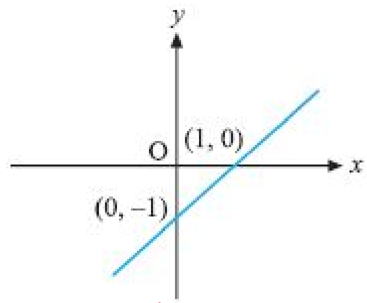
Ordered Pair : The combination of an input and output is called an ordered pair.

Representation of Ordered Pair : $\underset{\text{input}}{\underset{\downarrow}{(x}}\ ,\ \underset{\text{output}}{\underset{\downarrow}{y)}}$

So in above example orderd pairs are (0, 1), (–3, –2), (1, 2) etc., which satisfy the function:

1.2 Graph of a Function

It is the pictorial representation of a function. It is formed by plotting ordered pairs that satisfy function.



Let $y = f(x) = x - 1$. Graph of $y = f(x)$ is shown :

$y = f(x)$ is a linear polynomial whose graph is a straight line.

Note : A unique line passes through two given points. So to draw the graph of linear polynomials we needed to plot only two ordered pairs and join them.

Illustrating the Concept :

1. Suppose it is given that y is the square of x. Then we may write,

   $y = f(x) = x^2$

2. Velocity of uniformly accelerating particle starting from rest depends on time t.

   If acceleration is 2 m/s, we can write, velocity (v) as a function of t.

   i.e $v(t) = 0 + 2t$ **{ Using v = u + at }**

1.3 Intervals and Notations

To express values a variable can take, we use following notations.

(i) Open interval :

If x can take values which lie strictly between a and b then we can write

$a < x < b$ or $x \in (a, b)$

(ii) Closed interval :

If x can take values which lie strictly between a and b or x can be equal to a or x can be equal to b, then we can write

$a \leq x \leq b$ or $x \in [a, b]$

(iii) Half-open interval :

If only one end point is included for values of x, then the interval is called as half-open interval.

$a < x \leq b$ or $x \in (a, b]$

	(iv)	odd ÷ odd = even	
(e)	**(i)**	if $f(x) + f(-x) = 0$	$\Rightarrow$ f is odd function
	(ii)	if $f(x) - f(-x) = 0$	$\Rightarrow$ f is even function

(f) The square of an even or an odd function is always an even function.

2.4 Periodic function :

A function $f(x)$ is said to be periodic function of x, if there exists a positive real number T such that $f(x + T) = f(x)$.

The smallest value of T is called the period of the function.

Note : The positive T should be independent of x for $f(x)$ to be periodic. In case T is not independent of x, $f(x)$ is not a periodic function.

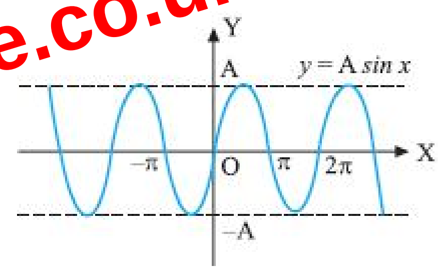
Definition (Graphically)

A function is said to be periodic if its graph repeats itself after a fixed interval and the width of that interval is called its period.

For example :

$\because$ Graph of $f(x) = A\sin x$ repeats after an interval of 2π.

Thus, $f(x) = A\sin x$ is periodic with period 2π.



Standard Results on Periodic Functions

	Function	Period
1.	$\sin^n x, \cos^n x$ $\sec^n x, \operatorname{cosec}^n x$	π, If n is even 2π, If n is odd or fraction
2.	$\tan^n x, \cot^n x$	π, n is even or odd
3.	$\lvert \sin x \rvert, \lvert \cos x \rvert$ $\lvert \tan x \rvert, \lvert \cot x \rvert$ $\lvert \sec x \rvert, \lvert \operatorname{cosec} x \rvert$	π π π
4.	$x - [x] = \{x\}$	1
5.	$\sqrt{x},\ x^2,\ x^3 + 5$ etc.	These function are not periodic

Properties of Periodic Function

(i) If $f(x)$ has period T, then

- **(a)** $cf(x)$ has also period T
- **(b)** $f(x + c)$ is periodic with period T

Continuity :

The graph of $y = A\sin(mx)$ and $y = A\cos(mx)$ is continuous (*i.e.* no break in the curve) every where.

The graph of $y = A\tan(mx)$ is discontinuous

(*i.e.* break in the curve) at $x = (2n+1)\dfrac{\pi}{2m}$

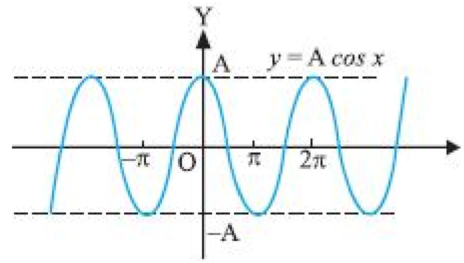
Domain and Range :

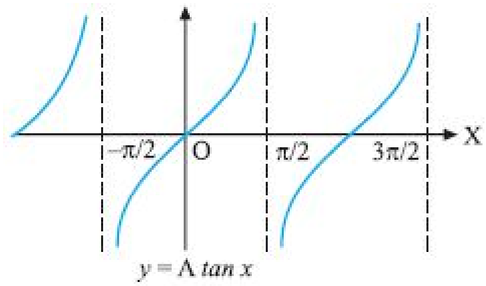
The Domain of $y = A\sin(mx)$ and $y = A\cos(mx)$ is $x \in R$

and Range is $y \in [-A, A]$.

The Domain of $y = A\tan(mx)$ is

$x \in R - (2n+1)\dfrac{\pi}{2m}$ and Range is $y \in R$.





(iv) $y = A\cot mx$

Period :

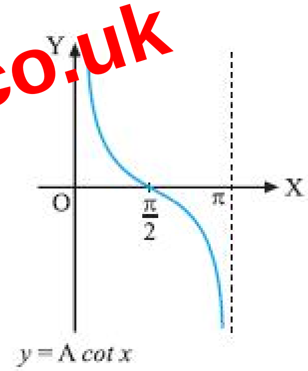
It is a periodic function with period = π/m

Continuity :

It can be observe that $y = A\cot mx$ is discontinuous at $x = n\pi/m$ where $n \in I$

Domain and Range :

The domain of $y = A\cot mx$ is $x \in R - \dfrac{n\pi}{m}$ and the range is $y \in R$.



(v) $y = A\sec mx$

Period :

It is a periodic function with period = $2\pi/m$.

Continuity :

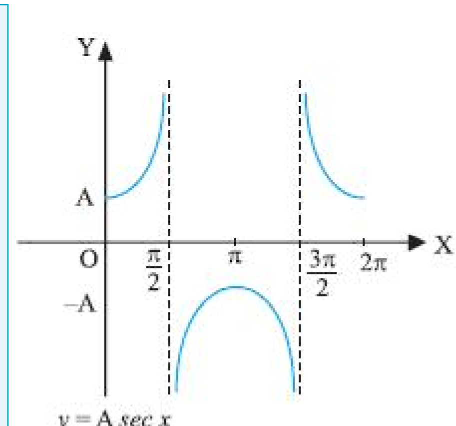
It can be observe that $y = A\sec mx$ is discontinuous at $x = (2n+1)\pi/2m$, $n \in I$

Domain and Range :

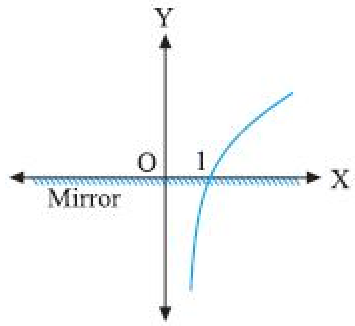
The domain of $y = A\sec mx$

is $x \in R - (2n+1)\dfrac{\pi}{2m}$

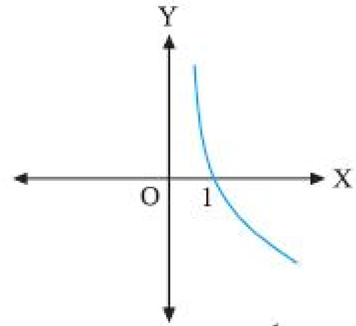
and the Range is $y \in (-\infty, -A] \cup [A, \infty)$.



(ii) $y = \log\frac{1}{x} = -\log x \xleftarrow{f(x) \to -f(x)} y = \log x$



Graph of $y = \log_e x$



Graph of $y = \log\frac{1}{x}$

Take mirror image about x-axis

(v) Transformation 5

$$y = f(x) \longrightarrow y = f(|x|)$$

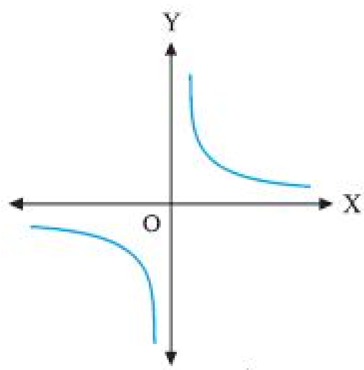
To plot $y = f(|x|)$, draw the graph of $y = f(x)$ first, then remove the portion of the graph in left half and after that take the mirror image of portion of the graph in right half in the Y-axis. Also include the right portion of the graph of $y = f(x)$.

Illustrating the concept :

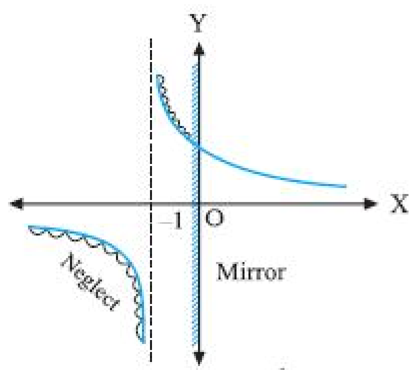
Sketch the graph of following functions :

(i) $y = \frac{1}{|x|+1}$; *(ii)* $y = x^2 - 2|x| + 3$

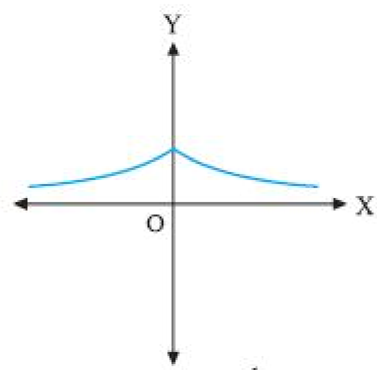
(i) $y = \frac{1}{|x|+1} \xleftarrow{f(x) \to f(|x|)} y = \frac{1}{x+1} \xleftarrow{f(x) \to f(x+a)} y = \frac{1}{x}$



Graph of $y = \frac{1}{x}$



Graph of $y = \frac{1}{x+1}$

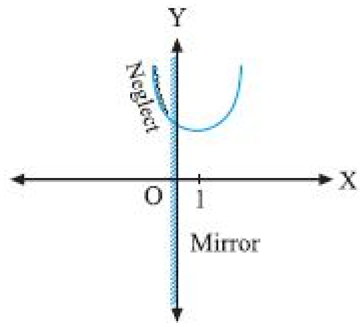


Graph of $y = \frac{1}{|x|+1}$

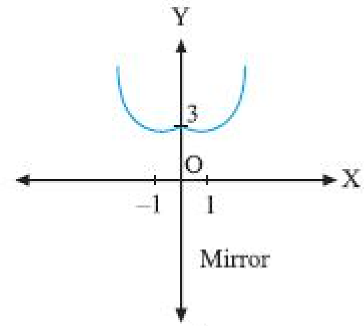
Shift 1 unit left

Reject left half and take reflection of right half in left half

(ii) $y = x^2 - 2|x| + 3 \xleftarrow{f(x) \to f(|x|)} y = x^2 - 2x + 3$



Graph of $y = x^2 - 2x + 3$



Graph of $y = x^2 - 2|x| + 3$

Reject left half and take reflection of right half in left half

(vi) Transformation 6

$$y = f(x) \longrightarrow y = |f(x)|$$

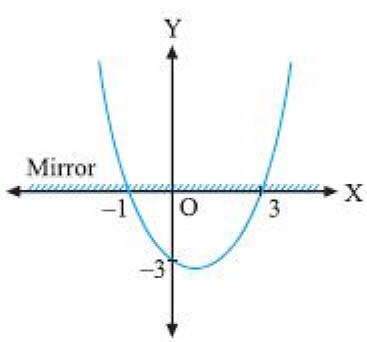
To plot $y = |f(x)|$, draw the curve $y = f(x)$, then take the mirror image of the lower portion of the curve (the curve below x-axis) in x–axis and then reject the lower part (or flip lower part into upper) Also include the upper protion of the curve $y = f(x)$.

Illustrating the concept :

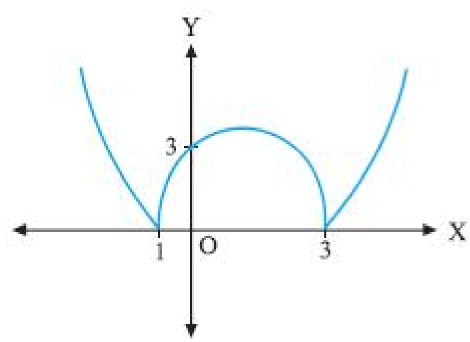
Draw the graph of the following curves :

(i) $y = |x^2 - 2x - 3|$; *(ii)* $y = |\log x|$.

(i) $y = |x^2 - 2x - 3| \xleftarrow{f(x) \to |f(x)|} y = x^2 - 2x - 3$



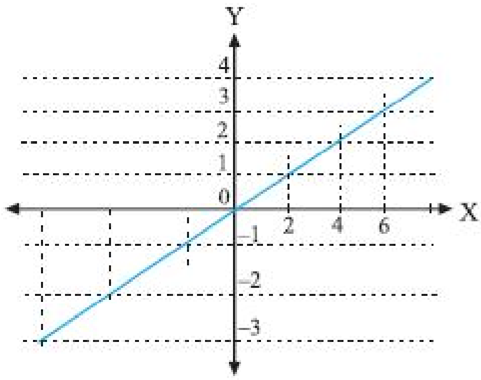
Graph of $y = x^2 - 2x - 3$



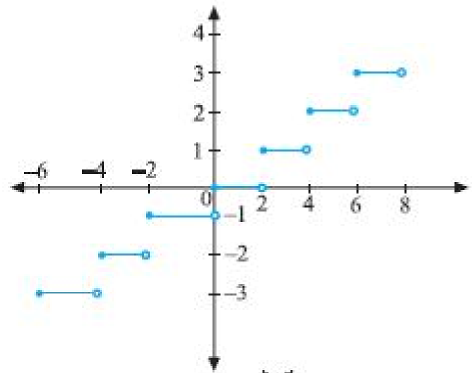
Graph of $y = |x^2 - 2x - 3|$

Flip lower part into upper

(ii) $y = [x/2]$



Graph of $y = \frac{x}{2}$



Graph of $y = \left[\frac{x}{2}\right]$

Draw horizontal line at $y \in I$

From each intersection point draw horizontal lines upto nearest right vertical line such that the horizontal line is always below the graph.

COMPOSITE FUNCTION — Section - 6

Let A, B and C be three non-empty sets.

Let $f : A \to B$ and $g : B \to C$ be two functions then $gof : A \to C$. This function is called **Composite Function.**

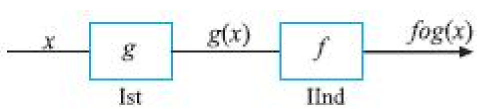
Note :

(i) $fog(x) = f(g(x))$; **(ii)** $fof(x) = f(f(x))$;

(iii) $gog(x) = g(g(x))$; **(iv)** $gof(x) = g(f(x))$.

Explanation :

(i) To undrestand the concept of composite function consider $fog(x)$:



In the above diagram, for Ist block, 'x' is the independent variable and corresponding $g(x)$ is the dependent variable. But for IInd block, $g(x)$ *i.e.* the dependent variable of Ist block is independent variable corresponding $fog(x)$ is the dependent variable.

Illustration - 9 *The domain of the function* $f(x)=\dfrac{1}{\sqrt{[x]^2-[x]-6}}$ is $x\in$:

(A) $(-\infty,-2)\cup[4,\infty)$ **(B)** $(-\infty,-2]\cup[4,\infty)$

(C) $(-\infty,-2)\cup(4,\infty)$ **(D)** *None of these*

SOLUTION : (A)

$f(x)$ is defined for

$[x]^2-[x]-6>0 \Rightarrow ([x]-3)\,([x]+2)>0$

$\Rightarrow\ [x]<-2$ or $[x]>3$

But $[x]<-2$ for $[x]=-3,-4,-5,\ldots$

$\therefore\quad x<-2$

Also $[x]>3$ for $[x]=4,5,6\ldots$

$\therefore\quad x\geq 4$

Domain of f is $x\in(-\infty,-2)\cup[4,\infty)$.

Illustration - 10 *The range of the function* $y=\dfrac{x}{1+x^2}$ is $y\in$:

(A) $\left[0,\frac{1}{2}\right]$ **(B)** $\left[-\frac{1}{2},\frac{1}{2}\right]$ **(C)** $\left[-\frac{1}{2},0\right]$ **(D)** *None of these*

SOLUTION : (B)

Clearly, y is defined for all real x.

$\therefore\quad$ Domain of y is $x\in(-\infty,\infty)$.

We have, $y=\dfrac{x}{1+x^2}\Rightarrow y+x^2y=x$

$\Rightarrow\quad x^2y-x+y=0$

For x to be real, discriminant of above quadratic equation ≥ 0

$1-4y^2\geq 0,\ y\neq 0$

$\Rightarrow\quad (1-2y)\,(1+2y)\geq 0,\ \ y\neq 0$

$\Rightarrow\quad (2y-1)\,(2y+1)\leq 0,\ y\neq 0$

$\Rightarrow\quad -\dfrac{1}{2}\leq y\leq\dfrac{1}{2},\ \ y\neq 0$

Also, if $y=0$, we get $x=0$ so that $y=0$ is also in the rnge.

$\therefore\quad$ Range of $y\in\left[-\dfrac{1}{2},\dfrac{1}{2}\right]$

Illustration - 11 The range of the function $y=\dfrac{x^2}{1+x^2}$ is $y\in$:

(A) $[0,1)$ **(B)** $(0,1)$ **(C)** $[0,1]$ **(D)** *None of these*