$$3.\int\frac{1-sinx}{cos^2x}dx$$

$$=\int\left(\frac{1}{cos^2x}-\frac{sinx}{cos^2x}\right)dx \quad \left[\because sec\alpha=\frac{1}{cos\alpha}\right]$$

$$=\int(sec^2x-secx.tanx)dx \quad \left[\because \int secx.tgxdx=secx+c\right]$$

$$=tanx-secx+c$$

$$\therefore \boxed{\int\frac{1-sinx}{cos^2x}dx=tanx-secx+c}$$

$$4.\int sin^2\frac{x}{2}dx$$

$$=\frac{1}{2}\int 2sin^2\frac{x}{2}dx$$

$$=\frac{1}{2}\int(1-cosx)dx \quad \left[\because 1-cos\alpha=2sin^2\alpha/2\right]$$

$$=\frac{x}{2}-\frac{sinx}{2}+c.$$

$$\because \boxed{\int sin^2\frac{x}{2}dx=\frac{1}{2}(x-sinx)+c.}$$

$$16.\ \int \frac{\sin^3 x-\cos^3 x}{\sin^2 x.\cos^2 x}dx$$

$$=\int \frac{\sin^3 x}{\sin^2 x.\cos^2 x}dx-\int \frac{\cos^3 x}{\sin^3 x.\cos^2 x}dx$$

$$=\int \frac{\sin x}{\cos^2 x}dx-\int \frac{\cos x}{\sin^3 x}dx$$

$$=\int tgx.\sec x\,dx-\int cosecx.tgx\,dx$$

$$=\sec x+cosecx+c$$

$$\therefore \boxed{\int \frac{\sin^3 x-\cos^3 x}{\sin^2 x.\cos^2 x}dx=\sec x+cosecx+c.}$$

$$17.\int\frac{\cos^2x-\sin^2x}{\sqrt{1+\cos4x}}dx$$

$$=\int\frac{\cos2x}{\sqrt{1+\cos4x}}dx$$

$$=\int\frac{\cos2x}{\sqrt{2\cos^22x}}dx$$

$$=\frac{1}{\sqrt{2}}\int\frac{\cos2x}{\cos2x}dx$$

$$=\frac{1}{\sqrt{2}}\int dx$$

$$=\frac{1}{\sqrt{2}}x+c$$

$$\therefore \boxed{\int\frac{\cos^2x-\sin^2x}{\sqrt{1+\cos4x}}dx=\frac{x}{\sqrt{2}}+c.}$$

$$18.\int\frac{3\cos x+4}{\sin^2x}dx$$

$$=3\int\frac{\cos x}{\sin^2x}dx+4\int\frac{dx}{\sin^2x}$$

$$=3\int cosec x.\cot x+4\int cosec^2x dx$$

$$=-3cosec x-4\cot x+c$$

$$\therefore \boxed{\int\frac{3\cos x+4}{\sin^2x}dx=-3cosec x-4\cot x+c.}$$

$$23. \int \frac{secx}{secx+tgx}dx$$

$$=\int \frac{secx}{secx+tgx}.\frac{(secx-tgx)}{(secx-tgx)}dx$$

$$=\int \frac{sec^2x-secx.tgx}{sec^2x-tg^2x}dx$$

$$=\int (sec^2x-secx.tgx)dx\ ;\ [\because sec^2a-tg^2a=1]$$

$$=tgx-secx+c.$$

$$\therefore \boxed{\int \frac{secx}{secx+tgx}dx=tgx-secx+c.}$$

$$24. \int \frac{cosx-cos2x}{1-cosx}dx$$

$$=\int \frac{cosx-(2cos^2x-1)}{1-cosx}dx$$

$$=\int \frac{cosx-2cos^2x+1}{1-cosx}dx$$

$$=-\int \frac{2cos^2x-cosx-1}{1-cosx}dx$$

$$=-\int \frac{(2cosx+1)(cosx-1)}{-(cosx-1)}dx$$

$$=\int (2cosx+1)dx$$

$$=2sinx+x+c$$

$$\therefore \boxed{\int \frac{cosx-cos2x}{1-cosx}dx=2sinx+x+c.}$$

28. $\int \frac{sin^6x+cos^6x}{sin^2x.cos^2x}dx$;[$a^3+b^3=(a+b)^3-3ab(a+b)$]

$$=\int \frac{(sin^2x+cos^2x)^3-3sin^2x.cos^2x(sin^2x+cos^2x)}{sin^2x.cos^2x}dx$$

$$=\int \frac{1-3sin^2x.cos^2x}{sin^2x.cos^2x}dx$$

$$=\int \frac{dx}{sin^2x.cos^2x}-3\int dx$$

$$=\int \frac{sin^2x+cos^2x}{sin^2x.cos^2x}dx-\int 3dx$$

$$=\int \frac{dx}{cos^2x}+\int \frac{dx}{sin^2x}-\int 3dx$$

$$=\int (sec^2x)dx+\int cosec^2x dx-\int 3dx$$

$$=tgx-cotx-3x+c.$$

$$\therefore \boxed{\int \frac{sin^6x+cos^6x}{sin^2x.cos^2x}dx=tgx-cotx-3x+c.}$$

$$38.\int cos^3x dx$$

$$where: cos3a = 4cos^3a - 3cosa$$

$$4cos^3a = cos3a + 3cosa$$

$$cos^3a = \frac{1}{4}(cos3a + 3cosa)$$

$$\because \int cos^3x dx$$

$$= \frac{1}{4}\int(cos3x + 3cosx)dx$$

$$= \frac{1}{4}\left(\frac{1}{3}sin3x + 3sinx\right) + c$$

$$= \frac{sin3x}{12} + \frac{3sinx}{4} + c.$$

$$\therefore \boxed{\int cos^3x dx = \frac{sin3x}{12} + \frac{3sinx}{4} + c.}$$

64. $\int \frac{2cosx-3sinx}{6cosx+4sinx}dx$

let $m=6cosx+4sinx$

$\Rightarrow dm=(4cosx-6sinx)dx$

$\Leftrightarrow \frac{dm}{2}=(2cosx-3sinx)dx$

$\because \int \frac{2cosx-3sinx}{6cosx+4sinx}dx$

$=\frac{1}{2}\int \frac{dm}{m}$

$=\frac{1}{2}ln|m|=\frac{1}{2}ln|(6cosx+4sinx)|+c$

$\therefore \boxed{\int \frac{2cosx-3sinx}{6cosx+4sinx}dx=\frac{ln|(6cosx+4sinx)|}{2}+c.}$

65. $\int \frac{sinx}{(1+cosx)^2}dx$

put $u=1+cosx \Rightarrow du=-sinxdx$

$\because \int \frac{sinx}{(1+cosx)^2}dx$

$=-\int \frac{du}{u^2}=\frac{1}{u}+c$

$=\frac{1}{1+cosx}+c$

$\therefore \boxed{\int \frac{sinx}{(1+cosx)^2}dx=(1+cosx)^{-1}+c.}$

$$72. \int \frac{sin2x}{sin4x}dx$$

$$=\int \frac{sin2x}{2sin2xcos2x}dx$$

$$=\frac{1}{2}\int \frac{dx}{cos2x}$$

$$=\frac{1}{2}\int sec2xdx$$

$$=\frac{1}{2}.\frac{ln|sec2x+tg2x|}{2}+c$$

$$\therefore \boxed{\int \frac{sin2x}{sin4x}dx=\frac{1}{4}ln|sec2x+tg2x|+c.}$$

$$81. \int \frac{\sin 2x - \cos x}{\sin^2 x - \sin x + 3} dx$$

$$= \int \frac{2\sin x . \cos x - \cos x}{\sin^2 x - \sin x + 3} dx$$

$$= \int \frac{(2\sin x - \cos x)}{\sin^2 x - \sin x + 3} . \cos x dx$$

$$let: u = \sin x \Rightarrow du = \cos x dx$$

$$= \int \frac{2u - 1}{u^2 - u + 3} du$$

$$= \int \frac{(u^2 - u + 3)'}{(u^2 - u + 3)} du$$

$$= \ln|u^2 - u + 3| + c.$$

$$= \ln|\sin^2 x - \sin x + 3| + c.$$

$$\therefore \boxed{\int \frac{\sin 2x - \cos x}{\sin^2 x - \sin x + 3} dx = \ln|\sin^2 x - \sin x + 3| + c.}$$