If the two displacement are in opposite direction, the instantaneous resultant displacement due to two waves acting together is expressed by

$$R_1 = x_1 - x_2$$

Interference is one of the effects of superposition of waves, in which two waves superimpose to from alternate maxima and minima.

1.7 Theory of Interference

If a_1 and a_2 are the amplitude of the two waves, the displacement due to one wave at any instant t is represented by

$$y_1 = a_1 \sin\omega t$$

and

$$y_2 = a_2 \sin(\omega t+\delta)$$

Where δ is the phase difference between two waves.

But according to the principle of super position

$$Y = y_1+y_2$$

$$Y = a_1\sin\omega t + a_2\sin(\omega t+\delta)$$

$$Y = a_1\sin\omega t + a_2\sin\omega t\cos\delta + a_2\cos\omega t\sin\delta$$

$$Y = (a_1 + a_2 \cos\delta)\sin\omega t + a_2 \sin\delta\cos\omega t$$

Let

$$a_1 + a_2 \cos\delta = R \cos\theta$$

$a_2 \sin\delta = R \sin\theta$ so above equation becomes

$$Y = R(\sin\omega t\cos\theta+\cos\omega t\sin\theta)$$

$$Y = R\sin(\omega t +\theta)$$

Squaring and adding, we get

$$R^2 = (a_1 + a_2 \cos\delta)^2 + a_2^2 \sin^2\delta$$

$$R^2 = a_1^2+a_2^2+2a_1a_2\cos\delta$$

But $I \alpha R^2$

So above equation can be written as

$$I = a_1^2+a_2^2+2a_1a_2\cos\delta$$

Or

$2\mu t \cos r = n\lambda$

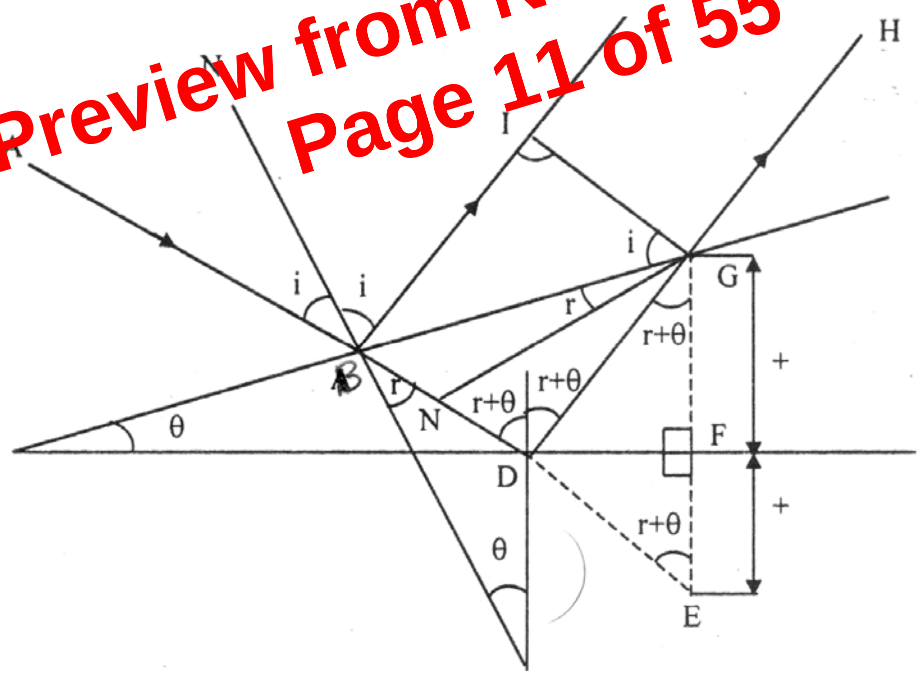
for destructive interference

$2\mu t \cos r = (2n - 1)\lambda/2$

1.11WEDGE-SHAPED FILM:

Let us take a wedge-shaped film (film of variable thickness) having angle of wedge θ. Suppose a beam of monochromatic light having wavelength λ incident on the upper surface of the film at an angle i at B. It gets partly reflected along BC and partly refracted along BD. At D again it gets partly reflected along DG & partly refracted. At again, it gets partly reflected and refracted along GH. The waves BC and GH are moving very close to each other and hence in a position to interfere. Similarly interference takes place in transmitted system.

The effective path difference between BC and GH will be given by



If D_n is the diameter of nth bright ring, we have $r_n = \frac{D_n}{2}$; therefore equation (4) becomes]

$$\left(\frac{D^2{}_n}{2}\right)^2 = \frac{(2n-1)\lambda R}{2\mu} \ldots\ldots$$

$$D^2{}_n = 2\frac{(2n-1)\lambda R}{\mu} \ldots\ldots\ldots\ldots\ldots\ldots(5a)$$

For air film $\mu = 1$

Therefore

$$D^2{}_n = 2(2n-1)\lambda R \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(5\ b)$$

Or

$D_n = \sqrt{[2\lambda R(2n-1)]} = \sqrt{(2\lambda R)}\sqrt{(2n-1)}$

$D_n = K\sqrt{(2n-1)}$. Where $K = \sqrt{(2\lambda R)}$………………………………(6)

As n = 1,2,3…………………(2n-1) is an odd number

Thus the diameter of the bright rings are proportional to the square roots of the odd natural numbers

FOR DARK RINGS:

Substituting value of t from3 in equation 2 we get

$$2\mu.\frac{r^2}{2R} = n\lambda \text{ or } r_n^2 = n\lambda R/\mu \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(7)$$

If D_n^2 is the diameter of nth bark rings, we have $r_n = D_n/2$,

Therefore equation (7) gives

$$D_n^2 = 4n\lambda R/\mu \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(8a)$$

For air film $\mu = 1$

$D_n^2 = 4n\lambda R$

Or $D_n = \sqrt{[4n\lambda R]} = \sqrt{(4\lambda R)}.\sqrt{n}$……………………………………………(8)

$$D_n^2 = 4n\lambda R/\mu \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

Similarly for (n+p)th ring

$$D^2_{n+p} = \frac{4(n+p)\lambda R}{\mu} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

From (1) and (2), we get

$$[D^2_{n+p} - D_n^2]_{liquid} = \frac{4p\lambda R}{\mu} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3)$$

Forair film, $\mu = 1$, therefore

$$[D^2_{n+p} - D_n^2]_{air} = \frac{4p\lambda R}{} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4)$$

Dividing (4) by (3) we get

$$\mu = \frac{[D_{n+p}^2 - D_n^2]_{air}}{[D_{n+p}^2 - D_n^2]_{liquid}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(5)$$

This relation also holds for bright rings

1.15 MICHELSON'S INTERFEROMETER:

PRINCIPLE:

The amplitude of light beam from a source is divided into two parts of equal intensities by partial reflection and transmission. These beams are then sent in two directions at right angles and are brought together after they suffer reflection from plane mirrors to produce interferences fringes.

$$\frac{I_{max}}{I_{min}} = \frac{(a_1 + a_2)^2}{(a_1 - a_2)^2}$$

$$\frac{I_1}{I_2} = \frac{a_1^{\,2}}{a_2^{\,2}}$$

$$\frac{a_1}{a_2} = \frac{9}{1} = a_1 = 9a_2$$

Substituting this value of a_1 in equation (1) we get

$$\frac{I_{max}}{I_{min}} = \left(\frac{9a_2 + a_2}{9a_2 - a_2}\right)^2 = \left[\frac{10}{8}\right]^2 = \frac{100}{64} = \frac{25}{16}$$

$$I_{max} : I_{min} = 25 : 16$$

Ex.2.Two coherent source of intensity ratio β interfere. Prove that in intensity pattern

$$\frac{I_{max} - I_{min}}{I_{max} - I_{min}} = \frac{2\sqrt{\beta}}{1+\beta}$$

sol.

$$I_{max} = (a_1 + a_2)^2$$

$$I_{min} = (a_1 - a_2)$$

$$\frac{I_1}{I_2} = \frac{a_1^{\,2}}{a_2^{\,2}} = \beta$$

$$\frac{a_1}{a_2} = \sqrt{\frac{I_1}{I_2}} = \sqrt{\beta}$$

(c) In both above

(d) In none above

18. A thin slice is cut out of a glass cylinder along a plane parallel to its axis. The slice is placed on a flat glass plate as shown: the observed interference fringes from the combination will be

(a) Straight (b) circular (c) equally spaced (d) having fringe spacing which increases as we go outwards

19. Oil floating on water looks coloured due to interference of light. The approximation thickness of oil film for such effect to be visible is

(a)100 $\overset{0}{A}$ (b) 10,000 $\overset{0}{A}$ (c) 1 mm (d) 1 cm

20. Newton's rings are

(a) locus of points of equal thickness

(b) Locus of points of equal inclination

(c) locus of points of equal thickness and equal inclinations

(d) neither (a) nor (b)

21. In the Newtron's ring arrangement the diameter of rings formed is proportional to

(a) λ (b) λ^2 (c) $\sqrt{\lambda}$ (d) $1/\sqrt{\lambda}$

22. In the Newton's ring arrangement with air film in reflected light the diameter of nth ring is D_n. If the air is replaced by liquid film of refracted index μ, the diameter of nth fringe will be

(a) $\sqrt{\mu}$ Times (b) $1/\sqrt{\mu}$ times (c) μ times (d) $1/\mu$ times

13. With a biprism arrangement the central fringe is achromatic when white light is used. Explain this.
14. Is there any conservation of energy in the phenomenon of Interference? Explain.
15. Explain the terms: fringes of equal thickness and fringes of equal inclination.
16. Find the ratio of intensity at the center of a bright fringe in an interference pattern to the intensity at a point one-quarter of the distance between two fringes from the center.

Numerical:

1. Two coherent sources whose intensity ratio is 81: 1 produce interference fringes. Deduce the ratio of maximum to minimum intensity of the fringe system.

[Ans. 25:16]

2. In an interference pattern at a point we observe the 12th order maximum for λ_1=600 nm. What order will be visible here if the source were replaced by the light of wavelength λ_2=480 nm?

[Ans. 15]

3. A source of light emits two wavelengths λ_1=5896Å and λ_2=5890Å. Interference fringes are observed with a certain arrangement when the path of interfering beams is exactly equal. How much will the path difference have to be increased so that a bright fringe for λ_1 coincides with a dark fringe for λ_2.

[Ans. 0.145 mm]

Division of wave front and Bi-prism:

Long Answer Type Question:

1. What is a Fresnel's biprism ?
2. Define fringe width. On what factors the fringe width of fringes in Fresnel's biprism arrangement depend ?.
3. What happens when a very thin film of refracting material is placed in the path of one of the interfering waves?

1. What are Newton's rings? Why does the centre of Newton's ring appear dark in reflected light.
2. Explain why Newton's rings are circular but air-wedge fringes are straight.
3. What change will occur in interference pattern when a little water in introduced between the lens and plate in Newton's rings arrangement?
4. What are Newton's ring's. Prove that in reflected light diameters of bright rings are proportional to the square root of odd natural numbers, (ii) diameters of dark rings are proportional to the square root of natural numbers.
5. What change would you expect in interference pattern of Newton's rings, if a plane mirror replaces the transparent plate below the lens.
6. Describe Newton's ring experiment for measuring the wavelength of monochromatic light and give the necessary theory. What will happen if a little water is introduced between the lens and the plate?

Numericals:

1. A plano-convex lens of radius 3 m. is placed on an optically flat glass plate and is illuminated by monochromatic light. The diameter of 8th dark ring in the transmitted system is 0. 72 cm. Calculate the wavelength of light used.

[Ans. 5760 $\overset{0}{A}$]

2. In Newton's rings arrangement the diameter of nth and *(n* + 14)th rings are 4.2 mm. and 7.0 mm. respectively. Radius of curvature of plano-convex lens is 1 m. calculate the wavelength of light.

[Ans. 5.6 x10^{-7} *m*.]

3. A convex lens of radius 3.50 m. placed on a flat plate and illuminated by monochromatic light gives the 6th bright ring of diameter 0. 68 cm. Calculate the wavelength of light used.

[Ans. 6000 $\overset{0}{A}$]

16. Newton's rings formed with sodium light between a flat glass plate and a convex lens is viewed normally. What will be the order of the dark ring, which will have double the diameter of that of the 40^{th} dark ring?

[Ans. 160]

Michenson's Interferometer:

Numericals:

1. In a Michelson's Interferometer 200 fringes cross the field of view, when the movable mirror is displaced through 0.0589 mm. Calculate the wavelength of monochromatic light used.

[Ans. 5890 Å]

2. A transparent film of glass of refractive index 1.50 is introduced normally in the path of one of the interfering beams of a Michelson's Interferometer, which is illuminated with a light of wavelength 4800 Å. This causes 500 dark fringes to sweep across the field. Determine the thickness of the film.

[Ans. 0.024 cm]

3. Michelson's interferometer experiment is performed with a source, which consists of two wavelengths 4882 Å and 4886 Å. Through what distance does the mirror have to be moved between two portions of the disappearance of the fringes?

[Ans. 0.149 mm]

4. When a thin plate of glass of refractive index 1.5 is placed in the path of one of the interfering beams of Michelson's interferometer, a shift of fringes of sodium light is observed across the field of view. If the thickness of the plate is 0.018mm, calculate the wavelength of light used.

[Ans. 5400 Å]