

Obtain all zeroes of

(3)

$$p(x) = 3x^4 + 6x^3 - 2x^2 - 10x - 5$$

Since two zeros are $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$

$$\left(x - \sqrt{\frac{5}{3}}\right)\left(x + \sqrt{\frac{5}{3}}\right) = \left(x^2 - \frac{5}{3}\right) \text{ is a}$$

factor of $3x^4 + 6x^3 - 2x^2 - 10x - 5$.

So we divide the polynomial by $x^2 - \frac{5}{3}$

$$\begin{array}{r} x^2 - \frac{5}{3} \overline{) 3x^4 + 6x^3 - 2x^2 - 10x - 5} \\ \underline{3x^4 - 5x^2} \\ 6x^3 - 3x^2 - 10x - 5 \\ \underline{6x^3 - 10x} \\ 3x^2 - 5 \\ \underline{3x^2 - 5} \\ 0 \end{array}$$

$$\begin{aligned} 3x^4 + 6x^3 - 2x^2 - 10x - 5 &= \left(x^2 - \frac{5}{3}\right)(3x^2 + 6x + 3) \\ &= \left(x^2 - \frac{5}{3}\right)3(x^2 + 2x + 1) \\ &= 3\left(x - \frac{5}{3}\right)(x + 1)^2 \end{aligned}$$