Linear Sum of two Subspaces :-

Let W_1 and W_2 be two SubSpaces of the vector Space $V(F)$.

Then the linear Sum of the Subspace is denoted by W_1+W_2 is the Set of all Sum of $\alpha_1+\alpha_2$ Such that $\alpha_1 \in W_1, \alpha_2 \in W_2$

$$\therefore W_1+W_2 = \{\alpha_1+\alpha_2 / \alpha_1 \in W_1, \alpha_2 \in W_2\}$$

Theorem ⑥ :- If W_1 and W_2 are any two Subspaces a vector Space $V(F)$ then.

(i) W_1+W_2 is a SubSpace of $V(F)$.

(ii) $W_1 \subseteq W_1+W_2$ and $W_2 \subseteq W_1+W_2$

Proof :- (i) Let $\alpha_1, \beta_1 \in W_1$ ~~$+W_2$ and $\alpha_2, \beta_2 \in W_2$~~ then

Then $\alpha = \alpha_1+\alpha_2$ and $\beta = \beta_1+\beta_2$

where $\alpha_1, \beta_1 \in W_1$ and $\alpha_2, \beta_2 \in W_2$

If $a, b \in F$ then $a\alpha_1, b\beta_1 \in W_1$ ($\because W_1$ Subspace)

and $a\alpha_2, b\beta_2 \in W_2$ ($\because W_2$ SubSpace)

Now $a\alpha+b\beta = a(\alpha_1+\alpha_2)+b(\beta_1+\beta_2)$

$= (a\alpha_1+b\beta_1)+(a\alpha_2+b\beta_2) \in W_1+W_2$

$\therefore a, b \in F$ and $\alpha, \beta \in W_1+W_2 \Rightarrow a\alpha+b\beta \in W_1+W_2$.

Hence W_1+W_2 is a Subspace of $V(F)$.

(ii) $\alpha_1 \in W_1$ and $\bar{0} \in W_2 \Rightarrow \alpha_1+\bar{0} \in W_1+W_2$

$\therefore \alpha_1 \in W_1 \Rightarrow \alpha_1 \in W_1+W_2$

$W_1 \subseteq W_1+W_2$

Similarly $W_2 \subseteq W_1+W_2$

Hence $W_1 \subseteq W_2 \subseteq W_1+W_2$