Mortality Table Construction

Individual data based methods

1. Exact exposure – for each individual is the amount of time from entry into the study until leaving the study. Leaving the study may be by death, withdrawal, or termination of the study. Exposure is tabulated by age.
   Assume: the hazard rate is constant for each age.
   - e_j: exact exposure for age j
   - d_j: the number of deaths
   - The hazard rate at that age: $h_j = \frac{d_j}{e_j}$
   - The probability of death in the one-year age interval: $\hat{q}_j = 1 - e^{-\frac{d_j}{e_j}}$
   - If the length of the interval is n years starting at age x, ${}_n\hat{q}_j = 1 - e^{-n\frac{d_j}{e_j}}$
2. Actuarial exposure – the same as exact exposure except for those who die.
   - The estimate of the mortality rate for the one-year age interval: $\hat{q}_j = \frac{d_j}{e_j}$
   - The actuarial exposure method may be generalised to multi-year age intervals: ${}_n\hat{q}_j = n\frac{d_j}{e_j}$
3. Variance of estimators
   Using the delta method,

$$\widehat{Var}({}_n\hat{q}_j) = (1 - {}_n\hat{q}_j)^2 n^2 \left(\frac{d_j}{e_j^2}\right)$$

$$\widehat{Var}(\hat{q}_j) = \frac{\hat{q}_j(1-\hat{q}_j)}{\frac{e_j}{n}}, \quad where\ \hat{q}_j = n\frac{d_j}{e_j}$$

4. Interval-based methods
   P_j: the population at time c_j
   n_j: the number of new entrants in the interval (c_j, c_{j+1})
   w_j: the number of withdrawals in the interval (c_j, c_{j+1})
   d_j: the number of deaths in the interval $(c_j, c_{j+1}]$
   n_j^b: the number of new entrants at the beginning of an interval
   n_j^m: the number of new entrants during an interval, not at the beginning
   w_j^m: the number of withdrawals during an interval, not at the end
   w_j^e: the number of withdrawals at the end of an interval

$$P_j = P_{j-1} + n_{j-1} - d_{j-1} - w_{j-1} - withdrawers\ at\ time\ c_j + entrants\ at\ time\ c_j$$

$$P_j = P_{j-1} + n_{j-1}^m - d_{j-1} - w_{j-1}^m - w_{j-1}^e + n_j^b$$

5. Multiple decrement
   The estimators may be used to calculate decrement rates in multiple-decrement situations. The estimators are all associated single-decrement rate estimators. If the forces of decrement are constant between interval boundaries and all decrements are independent, then the maximum likelihood estimate of the decrement rate equals the exact exposure estimate.
6. Estimating transition intensities
   Assume that the transition intensity μ^{ij} is constant in an interval. Then maximum likelihood can be used to derive an estimate similar to the exact exposure estimate with individual data. μ^{ij} is estimated as the number of transitions from i to j, divided by the amount of time spent by all individuals in state i.

$$\hat{\mu}^{ij} = \frac{d^{ij}}{e^{(i)}}$$