Gross loss at issue:

- gross future loss = PV(f.benefits) + PV(f.expenses) PV(f.gross premiums)
- Equivalence principle:

$$E(gross\ future\ loss) = 0$$

E(f.gross premium) = EPV(f.benefits) + EPV(f.expenses)

 $APV(future \ premium) = APV(future \ benefits)$ 

- Notations:
  - G: gross premium
  - $\circ$   $\pi$ : premium
  - *e*: the level renewal expense
  - $\circ$   $e_f$ : the first year expense
  - *E*: settlement expense
  - *b*: the face amount
- 1. Mean
  - Example:

For a fully discrete 20-year term policy on (30) with face amount 100,000, you are given

- $A_{30:20} = 0.450$ I.
- $A_{30:\frac{1}{20}} = 0.400$ II.
- III. Expenses are in the following table:

<u>Inpenses are in the following ta</u>				
	Per premium		er policy	
First year	35%	0.0	10.00	
Renewal	3%		2.00	
	in a large of the second			

- IV.
- V.
- v = 0.96The annual gross promium i (40).  $_0L$  is the gross loss at issue random variable (12) te E[-L]. VI.
- VII.
- Calculate *E* 
  - 230.<u>20</u> = 13.75 0.04
  - $E\begin{bmatrix} 0\\ 0\end{bmatrix} = 100,000A^{1}_{30;\overline{120}} + (0.32 + 0.03\ddot{a}_{30;\overline{120}})G + 8 + 2\ddot{a}_{30;\overline{120}} 13.75G$  $= 100,000(0.\overline{45} - 0.4) + (0.32 + 13.75 \times 0.03)400 + 8 + 2 \times 13.75 - 13.75G$ = -171.5
- 2. Variance discrete (2 approaches to calculate: formula & first principles)
  - a. By formula For whole life and endowment insurances with level expenses past issue, the formulas will work even if there are higher expenses in the 1<sup>st</sup> year payable at the beginning of the year, but otherwise expenses should not vary by duration.
  - For fully discrete whole life insurance, the gross future loss is

$${}_{0}L = (b+E)v^{K_{x}+1} + (e_{f}-e) - (G-e)\ddot{a}_{[K_{x}+1]}$$
$$Var({}_{0}L) = ({}^{2}A_{x} - A_{x}^{2})(b+E + \frac{G-e}{d})^{2}$$

- Notes:
  - 1) If G is determined by the equivalence principle, expenses do not differ between first year and renewal, and there are no settlement expenses (E), then G - e = net premiumand the formula reduces to the formula for the variance of the future net loss.

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2) A similar formula applies for endowment insurance.

$$Var(_{0}L) = \left({}^{2}A_{x:\underline{n}} - \left(A_{x:\underline{n}}\right)^{2}\right)\left(b + \frac{\pi}{d}\right)^{2}$$
$$Var(_{0}L) = b^{2}\left[\frac{{}^{2}A_{x:\underline{n}} - \left(A_{x:\underline{n}}\right)^{2}}{\left(1 - A_{x:\underline{n}}\right)^{2}}\right]$$
if equivalence principle premium is used
$$Var(_{0}L) = \frac{q(1-q)}{q + {}^{2}i}$$
for whole life with equivalence principle and constant rate of mortality only