Notations / Definitions:

- ${}_tp_x^{ij}$: the probability that someone in state i at time x is in state j (may = i) at time (x+t).
- ${}_tp_x^{\overline{ii}}$: the probability that someone in state i at time x stays in state i continuously until time (x+t).
- ${}_tp_x^{\overline{ii}} \leq {}_tp_x^{ij}$
- ${}_tp_x = {}_tp_x^{\overline{00}} = {}_tp_x^{00}$
- ${}_tq_x = {}_tp_x^{01}$ (since it is impossible to reenter state 0)
- Markov Chain is a multiple-state model with the following property: the probability of leaving a state is not a function of the amount of time in the state.
- 2 assumptions for multiple state models:
  - The probability of two or more transitions in a small amount of time is very small => o(h). A function of h, f(h), is o(h) if when it is divided by h, it goes to 0 as h goes to 0.

$$f(h) = o(h) \leftrightarrow \lim_{h\to 0}\frac{f(h)}{h} = 0$$

  - ${}_tp_x^{ij}$: a differentiable function of t for all i, j and x.

Discrete Markov chains

1. The transition probabilities for a finite-state Markov chain can be arranged in a matrix, the transition matrix. In the matrix, entry ij is the probability of transferring from state i to state j. Use $P^{(t)}$ for the matrix at time t.
2. Consider single life mortality for someone age 35. There will be a matrix for every duration. A dead person remains dead, so the probability of transition from state 1 to state 0 is 0 and the probability of transition from state 1 to state 1 is 1. For an alive person, the probability of transition to state 1 is q_{35+t}, and the probability of transition to state 0 (remaining alive) is p_{35+t}.
   a. $P^{(t)} = \begin{pmatrix} p_{35+t} & q_{35+t} \\ 0 & 1 \end{pmatrix}$
3. Consider the accidental death model for someone age 35. On the first row, the first entry will be the probability of survival, the second entry the probability of death from accidental causes, and the third entry the probability of death from other causes. On the other rows, the state cannot change.
   a. $P^{(t)} = \begin{pmatrix} p_{35+t} & p_{35+t}^{01} & p_{35+t}^{02} \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$
4. Consider the disability income model. This model allows more transitions than the previous two models, and the matrix would look like:
   a. $P^{(t)} = \begin{pmatrix} p_{35+t}^{00} & p_{35+t}^{01} & p_{35+t}^{02} \\ p_{35+t}^{10} & p_{35+t}^{11} & p_{35+t}^{12} \\ 0 & 0 & 1 \end{pmatrix}$
5. Chapman-Kolmogorov equation:

$${}_kp_x^{ij} = \sum_{m=1}^{n} {}_lp_x^{im}\,{}_{k-l}p_{x+l}^{mj}$$

   - n is the number of states
   - l is any integer between 0 and k
   - it says that the probability of transiting from state i to state j in k steps = the sum of the probabilities of transitioning from state i to some intermediate state m in l steps followed by transitioning from state m to state j in the remaining (k-l) steps, summed up over all possible intermediate states m.

Markov Chains Continuous Probability

1. ${}_tp_x^{ij}$ is differentiable for all i and j.
2. $\mu_x^{ij} = \lim_{h\to 0}\frac{{}_hp_x^{ij}}{h}$ (the force of transition)
3. ${}_hp_x^{ij} = h\mu_x^{ij} + o(h)$
4. ${}_hp_x^{ij} = {}_hp_x^{\overline{ii}} + o(h)$