

Modified and Reserves and other Reserve Topics

Semicontinuous reserves – suppose premiums are payable annually but the benefit is payable at the moment of death. *semicontinuous reserve* = $\frac{i}{\delta} \times \text{fully discrete reserve}$

For an endowment insurance with uniform distribution of deaths assumed, the semicontinuous reserve can be decomposed into a term reserve and a pure endowment reserve (*term reserve* $\times \frac{i}{\delta}$)

Fractional durations – fully discrete and the net premium reserve at time $k + s$, k is an integer and $0 < s < 1$

- Formulas

$${}_{k+s}V = \frac{({}_kV + \pi_{k+1})(1+i)^s - b_{k+1} {}_sq_{x+k} v^{1-s}}{s p_{x+k}}$$

- Under UDD between integral ages

$${}_{k+s}V = \frac{({}_kV + \pi_{k+1})(1+i)^s - s b_{k+1} q_{x+k} v^{1-s}}{1 - s q_{x+k}}$$

- Traditional approximation method

$${}_{k+s}V = (1-s)({}_kV + \pi_{k+1}) + s {}_{k+1}V$$

- Example

For a fully discrete insurance of 1000 on (x), you are given:

- Deaths are uniformly distributed between integral ages.
- ${}_9V = 100$
- $q_{x+9} = 0.05$
- $i = 0.25$
- The net premium is 20

Calculate ${}_{9.5}V$ exactly and using the traditional approximation.
For the exact method, the recursion is

$$\begin{aligned} {}_{9.5}V &= \frac{({}_9V + \pi_{10})(1+i)^{0.5} - 0.5b_{10}q_{x+9}(1+i)^{-0.5}}{1 - 0.5q_{x+9}} \\ &= \frac{120(1.25^{0.5}) - 25(1.25^{-0.5})}{0.975} = 114.67 \end{aligned}$$

For the traditional method,

$$\begin{aligned} {}_{10}V &= \frac{({}_9V + \pi_{10})(1+i) - 1000q_{x+9}}{1 - q_{x+9}} \\ &= \frac{120(1.25) - 50}{0.95} = 105.2632 \end{aligned}$$

The traditional approximation is ${}_{9.5}V = 0.5(100 + 20 + 105.2632) = 112.63$

Modified reserves

- In US, companies hold reserves based on net premium calculations. In order to lower the reserve to recognise expense, the reserve is modified. A lower modified net premium is used in the calculation in the 1st year, and higher modified net premiums are used in the later years, which is called modified reserve. Calculation method is Full Preliminary Term – FPT. In the FPT method, the modified net premium per unit – valuation premium or α . The renewal valuation premium is β .
- FPT is treated as if it is a one-year term insurance followed by an insurance issued to a life one year hold. For example, a whole life insurance on (55) would be treated as if it were a one-year term insurance on (55), followed by a whole life on (56). The reserve at the end of the 1st year is 0.
- expense allowance* = $\beta - \alpha$
 - Sometime the term “net premium” means the valuation premium.
 - Net premium computed using level premiums in all durations will be called “level net premium”.