Mortality Improvement Models

Deterministic Models

1. Single-factor projection scale – $\phi(x)$

$$q(x, y+t) = q(x, y)\big(1-\phi(x)\big)^t$$

- You are given the following mortality rates for 2020:

X	54	55	56	57
q_x	0.0032	0.0034	0.0037	0.0041

Projection scale AA for females is used. An excerpt is:

X	54	55	56	57
$\phi(x)$	0.01	0.008	0.006	0.005

Calculate the probability that a female age 54 in 2021 will die in the 3rd year.

- $q(54, 2021) = 0.0032(1-0.01) = 0.00317$
  $q(55, 2022) = 0.0034(1-0.008)^2 = 0.00335$ because (2022-2022) =2=t
  $q(56, 2023) = 0.0037(1-0.006)^3 = 0.00363$
  ${}_{2|}q(54, 2021) = (1-0.00317)(1-0.00335)(0.00363) = 0.003610$

2. Two-factor projection scale – $\phi(x, y)$

$$q(x, y) = q(x, y-1)\big(1-\phi(x, y)\big)$$

- $q_{80} = 0.0605$ for a female in 2014. Mortality improvement rates from MP-2014 are:

Year	2014	2015	2016	2017	2018
Age 80	0.0211	0.0203	0.0193	0.0183	0.0172

Calculate $q(80, 2018)$.

$$\begin{aligned} q(80, 2018) &= q(80, 2017)\big(1-\phi(80, 2018)\big) \\ &= q(80, 2016)\big(1-\phi(80, 2017)\big)\big(1-\phi(80, 2018)\big) \\ &= q(80, 2015)\big(1-\phi(80, 2016)\big)\big(1-\phi(80, 2017)\big)\big(1-\phi(80, 2018)\big) \\ &= q(80, 2014)\big(1-\phi(80, 2015)\big)\big(1-\phi(80, 2016)\big)\big(1-\phi(80, 2017)\big)\big(1 \\ &\quad -\phi(80, 2018)\big) \end{aligned}$$

$$q(80, 2018) = (0.0605)(1-0.0203)(1-0.0193)(1-0.0183)(1-0.0172) = 0.056083$$

- Conclusion for the two-factor projection scale formula:
  If the question gives the year in year A (MP-A), and the question is to calculate the mortality in year B.

$$q(x, y) = q(x, y-1)\big(1-\phi(x, y)\big)$$

$$q(x, B) = q(x, A)\big(1-\phi(x, A+1)\big)\big(1-\phi(x, A+2)\big)\ldots\big(1-\phi(x, B)\big)$$

3. Construction of MP-2014
   a. Develop short term improvement factors. Raw mortality experience from 1950-2007 is logged and smoothed in two dimensions to obtain $s(x, t)$. Then $\hat{q}(x, t) = e^{s(x,t)}$, and

$$\phi(x, t) = 1 - \frac{\hat{q}(x, t)}{\hat{q}(x, t-1)} = 1 - e^{s(x,t)-s(x,t-1)}$$