Inverse transformation : $q(x,t) = \frac{e^{lq(x,t)}}{1+e^{lq(x,t)}}$

b. Original CBD model

$$lq(x,t) = K_t^{(1)} + K_t^{(2)}(x-\overline{x})$$

- $\overline{x} =$ average age in the data set
- $K_t^{(1)}$ and $K_t^{(2)}$ are time series. We assume they are random walks with drift:
$$K_t^{(i)} = K_{t-1}^{(i)} + c^{(i)} + \sigma_{ki} Z_t^{(i)}, i = 1,2$$
- $Z_t^{(1)}$ and $Z_t^{(2)}$ are standard normal random variables with correlation ρ, but independent for different t.
- $E\left[Z_t^{(1)} Z_t^{(2)}\right] = \rho, -1 \leq \rho \leq 1$
- $E\left[Z_t^{(i)} Z_u^{(j)}\right] = 0, for\ t \neq u, i = 1,2; j = 1,2$
- Advantages: fewer parameters and less parameter uncertainty
- Disadvantage: fit to population is sometimes worse

c. CBD M7 model

$$lq(x,t) = K_t^{(1)} + K_t^{(2)}(x-\overline{x}) + K_t^{(3)}((x-\overline{x})^2 - s_x^2) + G_{t-x}$$

- Let $m = x_{max} - x_{min}$, then $s_x^2 = \frac{m(m+2)}{12}$, or $s_x^2 = \frac{\sum_{x=x_0}^{x_w}(x-\overline{x})^2}{x_w - x_0 + 1}$. s_x^2 is the variance of age range in the data set.
- G_{t-x} is the cohort effect. (ARIMA time series)
- Advantages: includes a cohort effect and a quadratic age difference term.