Multiple Decrements

Insurance and annuities

1. Suppose an insurance pays, for each decrement j, a benefit of $\mathrm{b}_{\mathrm{t}}^{(\mathrm{j})}$ at time t if decrement j occurs at that time. Then the net single premium for the insurance is

$$\bar{\mathrm{A}} = \int_0^\infty \mathrm{v}^{\mathrm{t}}\,{}_{\mathrm{t}}\,\mathrm{p}_{\mathrm{x}}^{(\tau)} \sum_{j=1}^{n} \mu_{x+t}^{(j)}\,\mathrm{b}_{\mathrm{t}}^{(\mathrm{j})}\mathrm{dt}$$

2. For an insurance payable at the end of the year of death:

$$\mathrm{A}_{\mathrm{x}}^{(\tau)} = \sum_{\mathrm{k}=0}^{\infty} \mathrm{v}^{\mathrm{k}+1}\,{}_{\mathrm{k}}\,\mathrm{p}_{\mathrm{x}}^{(\tau)} \sum_{j} b_j q_{x+k}^{(j)}$$

3. In any multiple decrement situation, each of the decrement is mutually exclusive, Z is the benefit random variable for an insurance paying $\mathrm{b}_{\mathrm{t}}^{(\mathrm{j})}$ at time t upon occurrence of decrement j, then

$$\mathrm{E[Z]} = \int_0^\infty v^t\,{}_t\,p_x^{(\tau)} \sum_{j=1}^{m} \mu_{x+t}^{(j)}\,b_t^{(j)}\mathrm{dt}$$

$$\mathrm{E[Z^2]} = \int_0^\infty v^{2t}\,{}_t\,p_x^{(\tau)} \sum_{j=1}^{m} \mu_{x+t}^{(j)}\left(b_t^{(j)}\right)^2 \mathrm{dt}$$

- Special formula for additional EPV of additional benefit $b^* = b^{(2)} - b^{(1)}$ paid for the first n years on secondary decrement with constant force $\mu^{(2)}$: $b^*\mu^{(2)}\bar{a}_{x:\overline{n|}}$

Examples

1. You are given the following for a three-decrement model:
   - I. $\mathrm{q}_{\mathrm{x}}^{\prime(1)} = 0.05$ and is uniformly distributed with each year of age.
   - II. $\mathrm{q}_{\mathrm{x}}^{\prime(2)} = 0.12$ and is uniformly distributed with each year of age.
   - III. $\mathrm{q}_{\mathrm{x}}^{\prime(3)} = 0.20$ and occurs at time 0.5 in each year.

   Determine $\mathrm{q}_{\mathrm{x}}^{(1)}$

$$q_x^{(1)} = (0.5)(0.05)\big(1 - 0.25(0.12)\big) + (0.5)(0.05)(1 - 0.2)\big(1 - 0.75(0.12)\big)$$
$$= 0.02425 + 0.0182 = 0.04245$$

2. In a double-decrement table:
   - I. $\mathrm{q}_{\mathrm{x}}^{\prime(1)} = 0.1$
   - II. Decrement (1) is uniformly distributed between integral ages in the associated single-decrement table.
   - III. ${}_tp_x^{\prime(2)} = \begin{cases} 1 - 0.2t & 0 < t < 1 \\ 0.5 & t = 1 \end{cases}$
   - IV. $l_x^{(\tau)} = 1000$

   Calculate $d_x^{(2)}$.

- As $t \to 1$, survival from decrement 2 approaches 0.8, but it then drops instantaneously to 0.5 at time 1, so decrement 2 has a point mass of 0.3 at 1; 30% of the population immediately before t=1 is decremented by it. Since the 2 decrements are linear before t=1, we can use the usual technique for calculating decrement 2's effect before 1: multiply the rate 0.2 by the average surviving from decrement 1, which is the midpoint, or $1 - 0.1(0.5) = 0.95$. The multiply the point mass of 0.3 at t=1 by the survivors from decrement 1 at that point, or $1 - 0.1 = 0.9$.

$$q_x^{(2)} = 0.2(0.95) + 0.3(0.9) = 0.46$$
$$d_x^{(2)} = 1000(0.46) = 460$$

3. A whole life insurance on 500 lives pays 1000 for deaths from any cause at any time, and an additional 9000 for deaths from accidental causes within the first 20 years only. Benefits are paid at the moment of death.