$$e_{xy:\overline{n}|} = \int_0^n {}_tp_{xy}\, dt$$

2. For the expected future lifetime of the last survivor status (the expected amount of time to the second death), we can use $e_{\overline{xy}} = e_x + e_y - e_{xy}$ (dependent or independent).
3. Two special cases: exponential lifetimes and uniform survival
   a. Uniform / Beta
      - If each life's future lifetime is beta with parameters ω_x and α_x for the first life, ω_y and α_y for the second life, and $\omega_x - x = \omega_y - y$ for the two lives, then the joint life status follows a beta distribution with $\alpha = \alpha_x + \alpha_x$. For a beta distribution with α and ω, expected future lifetime for (x) is $e_x = \frac{\omega - x}{\alpha + 1}$
      - If future survival for each of two lives (x) and (y) is uniform with limiting ages ω_x and ω_y, but $\omega_x - x \neq \omega_y - y$. Let $a = \omega_x - x$ and $b = \omega_y - y$. Then,

$$e_{xy} = E[T_{xy}] = {}_ap_y E[T_{xy} | T_Y > a] + {}_aq_y E[T_{xy} | T_Y \leq a]$$

      - If (y) survives to time a, T_{xy} depends only on the lifetime of (x) and is uniform on (0,a], so T_{xy}'s expected value is the midpoint of (0,a], $\frac{a}{2}$. If (y) does not survive to time a, the conditional lifetime of (y) given that (y) died before a is uniform on (0,a], so the joint lifetime of (x) and (y) given that (y) died before a is the joint lifetime of 2 uniform lives with the same maximum future lifetime or a beta distribution with $\alpha = 2$ and $\omega - x = \alpha$, and the expected value of T_{xy} is $\frac{a}{3}$. Then,

$$e_{xy} = {}_ap_y \left(\frac{a}{2}\right) + {}_aq_y \left(\frac{a}{3}\right)$$

Since ${}_aq_y = \frac{a}{b}$, the final formula is

$$e_{xy} = \left(1 - \frac{a}{b}\right)\left(\frac{a}{2}\right) + \left(\frac{a}{b}\right)\left(\frac{a}{3}\right) = \frac{a}{2} - \frac{a^2}{2b} + \frac{a^2}{3b} = \frac{a}{2} - \frac{a^2}{6b}$$

$$e_{\overline{xy}} = e_x + e_y - e_{xy} = \frac{a}{2} + \frac{b}{2} - \frac{a}{2} + \frac{a^2}{6b} = \frac{b}{2} + \frac{a^2}{6b}$$

   b. Exponential

$$e_{xy} = \int_0^\infty {}_tp_{xy}\, dt$$

$$e_{\overline{xy}} = \int_0^\infty {}_tp_{\overline{xy}}\, dt$$

$$e_{xy:\overline{n}|} = \int_0^n {}_tp_{xy}\, dt$$

4. To calculate the variance of future lifetime for joint life and last survivor statuses, we can use

$$Var(T_{xy}) = 2\int_0^\infty t \; {}_tp_{xy} dt - e_{xy}^2$$

5. Covariance formula:

$$Cov(T_{xy}, T_{\overline{xy}}) = Cov(T_x, T_y) + (e_x - e_{xy})(e_y - e_{xy})$$