Multiple lives

Contingent probabilities (recall)
Notation:

${}_tq^1_{xy}$: the probability that (x) dies first by time t before (y)

- To express the probability as an integral, integrate the probability of staying in 0 to time s followed by transfer to state 2:

$$ {}_tq^1_{xy} = \int_0^t {}_sp^{00}_{xy}\,\mu^{02}_{x+s:y+s}ds $$

${}_tq^2_{xy}$: the probability that (x) dies second by time t

- To express the probability as an integral, use a double integral for the two transitions:

$$ {}_tq^2_{xy} = \int_0^t {}_sp^{00}_{xy}\,\mu^{01}_{x+s:y+s}\int_0^{t-s} {}_up^{11}_{x+s}\,\mu^{13}_{x+s+u}\,du\,ds $$

Annuities

- Recall: $\overline{a}_x + \overline{a}_y = \overline{a}_{xy} + \overline{a}_{\overline{xy}}$
- 3 ways to calculate expected present values of annuities

1. Two-annuity technique: calculate 2 single annuities, then adjust the joint annuity. The first technique is calculating annuities for each life, then cancel or augment the payments for the joint life status when both are alive.
2. Three-annuity technique: calculate 2 single life annuities and 1 joint life annuity. The second technique is to calculate and add up the following 3 annuities:
   a. An annuity payable when only the first life is alive. This possibly require calculating an annuity payable when the first life is alive and subtracting from it an annuity payable when both lives are alive.
   b. An annuity payable when only the second life is alive. This may require calculating an annuity payable when the second life is alive and subtracting from it an annuity payable when both lives are alive.
   c. An annuity payable when both lives are alive.
3. Reversionary annuity technique: calculate a full annuity and cancel out the part that is not paid.
   a. A reversionary annuity is an annuity that makes regular payments to one status after another status has failed.
   b. IAN symbol: $a_{x|y}$, for the expected present value of the annuity
   c. $a_{x|y} = a_y - a_{xy}$
   d. If an annuity is paid to (z_2) after (z_1) expires, where (z_1) and (z_2) can be any combination of lives and certain statuses, then $\ddot{a}_{z_1|z_2} = \ddot{a}_{z_2} - \ddot{a}_{z_1:z_2}$
   e. It's possible for either status, (x) or (y), to include certain periods, or to consist entirely of a certain period. For example, $a_{x:\overline{\boxed{20}}|y}$ would pay 1 per year to (y) only after the later of 20 years and the death of (x).