

Operations with Algebraic and Rational Expressions

$$\frac{b-5}{5-b} = \frac{b-5}{-(b-5)} = -1$$

Factorization:

$$\frac{5}{-1} + \frac{-b}{-1} = -(-5+b)$$

Multiplying Rational expressions

 Let's learn how to do it:

$$\frac{5xy^3}{8x^2z^2} \times \frac{16z^3}{15y^2} = \frac{yz}{x^3}$$

$$\frac{5 \div 5}{15 \div 5} = \frac{1}{3}$$

$$\frac{16 \div 8}{8 \div 8} = \frac{2}{1}$$

$$\frac{y^3}{y^2} = y^{3-2} = y$$

$$\frac{x}{x^2} = x^{1-2} = x^{-1} = \frac{1}{x}$$

Examples:

$$\frac{x+3}{x} \times \frac{x^2}{x^2 - 2x - 15}$$

Identity IV

Find two numbers that:

$$3 \times (-5) = -15$$

$$3 + (-5) = -2$$

We rewrite expression:

$$\frac{x+3}{x} \times \frac{x^2}{(x+3)(x-5)} = \frac{x}{(x-5)}$$

factorization

$$\frac{x^2 + 5x}{x} \times \frac{1}{x^2 + 10x + 25} = \frac{x(x+5)}{x} \times \frac{1}{(x+5)^2} = \frac{1}{x+5}$$

Identity I

Operations with Algebraic and Rational Expressions

To add or subtract two rational expressions with unlike denominator

 Let's learn how to do it:

find the least common multiple (LCM) of the two denominators

$$\frac{a}{b} + \frac{c}{d} = \frac{ad + cb}{bd}$$

$(b, d \neq 0)$

$$\frac{1}{3a} + \frac{1}{4b} = \frac{1(4b)}{3a(4b)} + \frac{1(3a)}{4b(3a)} = \frac{4b + 3a}{12ab}$$

The LCD of the fraction is 12ab

$$\frac{1}{a-5} + \frac{2}{a} = \frac{1a + 2(a-5)}{a(a-5)} = \frac{a + 2a - 10}{a(a-5)} = \frac{3a - 10}{a(a-5)}$$

The LCM of the fraction is $a(a-5)$

Examples:

$$\frac{1}{x+1} - \frac{2}{x-1} = \frac{(x-1) + 2(x+1)}{(x+1)(x-1)} = \frac{(x-1) + 2x + 2}{x^2 - 1} = \frac{3x + 2}{x^2 - 1}$$

$$\frac{2}{x+2} + \frac{x-1}{x+4} = \frac{2(x+4) + (x-1)(x+2)}{(x+2)(x+4)}$$

Identity IV

$$\begin{aligned}
 &= \frac{2x + 8 + x^2 + (2-1)x + (-1 \times 2)}{x^2 + (2+4)x + (2 \times 4)} \\
 &= \frac{2x + 8 + x^2 + x - 2}{x^2 + 6x + 8} = \frac{x^2 + 3x + 6}{x^2 + 6x + 8}
 \end{aligned}$$