5. Find the (i) 6th term of $\left(1+\frac{x}{2}\right)^{-5}$.

Sol. T_{r+1} in $(1 + x)^{-n} = (-1)^r \frac{(n)(n+1)(n+2)...(n+r-1)}{1\cdot 2\cdot 3\cdot ...r}\cdot x^r$

Put r = 5, n = 5, x by x/2

$$T_6 = (-1)^5 \frac{(5)(5+1)(5+2)(5+3)(5+4)}{1\cdot 2\cdot 3\cdot 4\cdot 5}\cdot\left(\frac{x}{2}\right)^5$$

$$= \frac{-5\cdot 6\cdot 7\cdot 8\cdot 9}{1\cdot 2\cdot 3\cdot 4\cdot 5}\cdot\left(\frac{1}{2}\right)^5\cdot x^5 = \frac{-63}{16}\cdot x^5$$

ii) 7th term of $\left(1-\frac{x^2}{3}\right)^{-4}$

Sol. T_{r+1} in $(1 - x)^{-n}$ =

$$= \frac{(n)(n+1)(n+2)...(n+r-1)}{1\cdot 2\cdot 3\cdot ...r}\cdot x^r$$

Put r = 6, n = 4, x by $\frac{x^2}{3}$

Then 7th term in $\left(1-\frac{x^2}{3}\right)^{-4}$ is

$$= \frac{(4)(4+1)(4+2)(4+3)(4+4)(4+5)}{1\cdot 2\cdot 3\cdot 4\cdot 5\cdot 6}\left(\frac{-x^2}{3}\right)^6$$

$$= \frac{4\cdot 5\cdot 6\cdot 7\cdot 8\cdot 9}{1\cdot 2\cdot 3\cdot 4\cdot 5\cdot 6}\cdot\frac{x^{12}}{3^6} = \frac{28}{243}\cdot x^{12}$$

iii) 10th term of $(3 - 4x)^{-2/3}$.

Sol. $(3-4x)^{-2/3} = \left[3\left(1-\frac{4}{3}x\right)\right]^{-2/3} = (3)^{-2/3}\left(1-\frac{4}{3}x\right)^{-2/3}$ ...(1)

First find 10th term of $\left(1-\frac{4}{3}x\right)^{-2/3}$

The general term of $(1 - x)^{-p/q}$ is $T_{r+1} = \frac{(p)(p+q)(p+2q)+...+[p+(r-1)q]}{(r)!}\left(\frac{x}{q}\right)^r$

Here p = 2, q = 3, r = 9

We know that

$$(1-X)^{p/q} = 1 - p\left(\frac{X}{q}\right) + \frac{(p)(p-q)}{1\cdot 2}\left(\frac{X}{q}\right)^2 - \ldots$$

Here $X = \frac{5x}{8}, p = 2, q = 3, \frac{X}{q} = \frac{(5x/8)}{3} = \frac{5x}{24}$

$\therefore (8 - 5x)^{2/3} =$

$$4\left[1 - 2\left(\frac{5x}{24}\right) + \frac{(2)(2-3)}{1\cdot 2}\left(\frac{5x}{24}\right)^2 - \ldots\right]$$

$$= 4\left[1 - \frac{5x}{12} - \left(\frac{5x}{24}\right)^2 + \ldots\right]$$

$\therefore$ The first 3 terms of $(8 - 5x)^{2/3}$ are

$$4, \frac{-5x}{3}, \frac{-25}{144}x^2$$

iv) $(2 - 7x)^{-3/4}$ Try your self

7. Find the general term (r + 1)th term in the expansion of

(i) $(4 + 5x)^{-3/2}$ (ii) $\left(1 - \frac{5x}{3}\right)^{-3}$ (iii) $\left(1 + \frac{4x}{5}\right)^{5/2}$ (iv) $\left(3 - \frac{5x}{4}\right)^{-1/2}$

i) $(4 + 5x)^{-3/2}$

Sol. Write $(4 + 5x)^{-3/2} = \left[4\left(1 + \frac{5}{4}x\right)\right]^{-3/2}$

$$= (2^2)^{-3/2}\left[\left(1 + \frac{5}{4}x\right)^{-3/2}\right] = \frac{1}{8}\left[\left(1 + \frac{5}{4}x\right)^{-3/2}\right]$$

General term of $(1 + x)^{-p/q}$ is

$T_{r+1} = (-1)^r$

iii) x^2 in $\left(7x^3 - \frac{2}{x^2}\right)^9$ **Ans.** Coefficient of x^2 in $\left(7x^3 - \frac{2}{x^2}\right)^9$ is $-126 \times 7^4 \times 2^5$.

iv) x^{-7} in $\left(\frac{2x^2}{3} - \frac{5}{4x^5}\right)^7$

Sol. The general term in $\left(\frac{2x^2}{3} - \frac{5}{4x^5}\right)^7$ is

$$T_{r+1} = (-1)^r \cdot {}^7C_r \left(\frac{2x^2}{3}\right)^{7-r} \left(\frac{5}{4x^5}\right)^r$$

$$= (-1)^r \cdot {}^7C_r \left(\frac{2}{3}\right)^{7-r} \left(\frac{5}{4}\right)^r x^{14-2r} x^{-5r}$$

$$\therefore T_{r+1} = (-1)^r\, {}^7C_r \left(\frac{2}{3}\right)^{7-r} \left(\frac{5}{4}\right)^r x^{14-7r} \quad ...(1)$$

For coefficient of x^{-7}, put $14 - 7r = -7$

$\Rightarrow 7r = 21 \Rightarrow r = 3$

Put $r = 3$ in equation (1)

$$T_{3+1} = (-1)^3\, {}^7C_3 \left(\frac{2}{3}\right)^4 \left(\frac{5}{4}\right)^3 x^{14-21}$$

$$= \frac{-7 \times 6 \times 5}{1 \times 2 \times 3} \left(\frac{2}{3}\right)^4 \left(\frac{5}{4}\right)^3 x^{-7}$$

$\therefore$ Coefficient of x^{-7} in $\left(\frac{2x^2}{3} - \frac{5}{4x^5}\right)^7$ is:

$$= -35 \times \frac{1}{3^4} \cdot \frac{5^3}{2^2} = \frac{-4375}{324}$$

Its integral part $m = \left[11\frac{17}{29}\right] = 11$

T_{m+1} is the numerically greatest term in the expansion $\left(1+\frac{3}{4}x\right)^{15}$ and

$$T_{m+1} = T_{12} = {}^{15}C_{11}\left(\frac{3}{4}x\right)^{4} = {}^{15}C_{11}\left(\frac{3}{4}\cdot\frac{7}{2}\right)^{11}$$

$\therefore$ Numerically greatest term in $(4 + 3x)^{15}$

$$= 4^{15}\left[{}^{15}C_{11}\left(\frac{21}{8}\right)^{11}\right] = {}^{15}C_{4}\frac{(21)^{11}}{2^{3}}$$

ii) $(3x + 5y)^{12}$ when $x = \frac{1}{2}$ and $y = \frac{4}{3}$

Sol. Write $(3x + 5y)^{12} = \left[3x\left(1+\frac{5y}{3x}\right)\right]^{12}$

$$= 3^{12}x^{12}\left(1+\frac{5}{3}\frac{y}{x}\right)^{12}$$

On comparing $\left(1+\frac{5}{3}\frac{y}{x}\right)^{12}$ with $(1 + x)^{n}$, we get

$n = 17$, $x = \frac{5}{3}\cdot\frac{y}{x} = \frac{5}{3}\frac{(4/3)}{(1/2)} = \frac{5}{3}\cdot\frac{8}{3} = \frac{40}{9}$

Now $\frac{(n+1)|x|}{1+|x|} = \frac{(12+1)\left(\frac{40}{9}\right)}{1+\frac{40}{9}}$

$$= \frac{13\times 40}{49} = \frac{520}{49} = 10\frac{30}{49}$$

Which is not an integer.

$\therefore$ k = 11

Put r = 10 in eq.(1)

$$T_{13+1} = {}^{20}C_{13}(x^2)^7\left(\frac{-1}{2x}\right)^{13} = (-1)\,{}^{20}C_{13}\frac{1}{2^{13}}x$$

12. If the coefficients of (2r + 4)th and (r – 2)nd terms in the expansion of $(1 + x)^{18}$ are equal, find r.

Sol. T_{2r+4} term of $(1 + x)^{18}$ is

$$T_{2r+4} = {}^{18}C_{2r+3}(x)^{2r+3}$$

T_{r-2} term of $(1+x)^{18}$

$$T_{r-2} = {}^{18}C_{r-3}(x)^{r-3}$$

Given that the coefficients of (2r + 4)th term = The coefficient of (r – 2)nd term.

$\Rightarrow {}^{18}C_{2r+3} = {}^{18}C_{r-3}$

$\Rightarrow 2r+3 = r-3$ (or) $(2r+3)+(r-3) = 18$

$\Rightarrow r = -6$ (or) $3r = 18 \Rightarrow r = 6$

13. Find the coefficient of x^{10} in the expansion of $\frac{1+2x}{(1-2x)^2}$.

Sol. $\frac{1+2x}{(1-2x)^2} = (1+2x)(1-2x)^{-2}$

$= (1+2x)[1+2(2x)+3(2x)^2+4(2x)^3+5(2x)^4+6(2x)^5+7(2x)^6+8(2x)^7+9(2x)^8+10(2x)^9$
$+11(2x)^{10}+\ldots+(r+1)(2x)^r+\ldots]$

$\therefore$ The coefficient of x^{10} in $\frac{1+2x}{(1-2x)^2}$ is

$= (11)(2)^{10}+10(2)(2^9) = 2^{10}(11+10) = 2\times1^{10}$

$$= \frac{2}{9}\left(1+\frac{3}{4}x\right)^{1/2}\left(1-\frac{2}{3}\right)^{-2}$$

$$= \frac{2}{9}\left(1+\frac{1}{2}\cdot\frac{3}{4}x\right)\left(1-(-2)\frac{2}{3}x\right)$$

(After neglecting x^2 and higher powers of x)

$$= \frac{2}{9}\left(1+\frac{3}{8}x\right)\left(1+\frac{4}{3}x\right) = \frac{2}{9}\left(1+\frac{3}{8}x+\frac{4}{3}x\right)$$

(Again by neglecting x^2 term)

$$= \frac{2}{9}\left(1+\frac{41}{24}x\right) = \frac{2}{9}+\frac{41}{108}x$$

$$\therefore \frac{(4+3x)^{1/2}}{(3-2x)^2} = \frac{2}{9}+\frac{82}{108}x = \frac{2}{9}+\frac{41}{108}x$$

ii) $$\frac{\left(1-\frac{2x}{3}\right)^{3/2}(32+5x)^{1/5}}{(3-x)^3}$$

Sol. $$\frac{\left(1-\frac{2x}{3}\right)^{3/2}(32+5x)^{1/5}}{(3-x)^3}$$

$$= \frac{\left(1-\frac{2}{3}x\right)^{3/2}(32)^{1/5}\left(1+\frac{5}{32}x\right)^{1/5}}{3^3\left(1-\frac{x}{3}\right)^3}$$

$$= \frac{2}{27}\left(1-\frac{2x}{3}\right)^{3/2}\left(1+\frac{5}{32}x\right)^{1/5}\left(1-\frac{x}{3}\right)^{-3}$$

$$= \frac{2}{27}\left(1-\frac{3}{2}\cdot\frac{2x}{3}\right)\left(1+\frac{1}{5}\frac{5}{32}x\right)\left(1+3\frac{x}{3}\right)$$

(By neglecting x^2 and higher powers of x)

19. Suppose p, q are positive and p is very small when compared to q. Then find an approximate value of $\left(\frac{q}{q+p}\right)^{1/2}+\left(\frac{q}{q-p}\right)^{1/2}$

Sol. Do it yourself. Same as above.

20. By neglecting x^4 and higher powers of x, find an approximate value of $\sqrt[3]{x^2+64}-\sqrt[3]{x^2+27}$**.**

Sol. $\sqrt[3]{x^2+64}-\sqrt[3]{x^2+27}$

$$=(64+x^2)^{1/3}-(27+x^2)^{1/3}$$

$$=(64)^{1/3}\left(1+\frac{x^2}{64}\right)^{1/3}-(27)^{1/3}\left(1+\frac{x^2}{27}\right)^{1/3}$$

$$=4\left(1+\frac{x^2}{192}\right)-3\left(1+\frac{x^2}{81}\right)$$

(By neglecting x^4 and higher powers of x)

$$=4+\frac{x^2}{48}-3-\frac{x^2}{27}=1+\frac{(27-48)}{48\times 27}x^2$$

$$=1+\left(\frac{-21}{48\times 27}\right)x^2=1-\frac{7x^2}{432}=1-\frac{7}{432}x^2$$

$$\therefore \sqrt[3]{x^2+64}-\sqrt[3]{x^2+27}=1-\frac{7}{432}x^2$$

21. Expand $3\sqrt{3}$ in increasing powers of 2/3.

Sol. $3\sqrt{3}=3^{3/2}=\left(\frac{1}{3}\right)^{-3/2}=\left(1-\frac{2}{3}\right)^{-3/2}$

$$=1+\frac{\frac{3}{2}}{1}\cdot\left(\frac{2}{3}\right)+\frac{\frac{3}{2}\left(\frac{3}{2}+1\right)}{1\cdot 2}\left(\frac{2}{3}\right)^2+\ldots\ldots+\frac{\frac{3}{2}\left(\frac{3}{2}+1\right)\ldots\ldots\left(\frac{3}{2}+r-1\right)}{(1\cdot 2\cdot 3\ldots.r)2^r}\left(\frac{2}{3}\right)^r+\ldots\ldots$$

$$=1+\frac{3}{1\cdot 2}\left(\frac{2}{3}\right)+\frac{3\cdot 5}{(1\cdot 2)2^2}\left(\frac{2}{3}\right)^2+\ldots\ldots+\frac{3\cdot 5\ldots(2r+1)}{(1\cdot 2\cdot\ldots r)2^r}\left(\frac{2}{3}\right)^r+\ldots\ldots$$

25. Find the numerically greatest term(s) in the expansion of

i) $(2 + 3x)^{10}$ when $x = \frac{11}{8}$

Sol. Write $(2+3x)^{10} = \left[2\left(1+\frac{3}{2}x\right)^{10}\right] = 2^{10}\left(1+\frac{3x}{2}\right)^{10}$

First find N.G. term in $\left(1+\frac{3x}{2}\right)^{10}$

Let $X = \frac{3x}{2} = \frac{3\times\frac{11}{8}}{2} = \frac{33}{16}$

Now consider

$$\frac{(n+1)|x|}{1+|x|} = \frac{(10+1)\left(\frac{33}{16}\right)}{\frac{33}{16}+1} = \frac{11\times 33}{48} = \frac{363}{48}$$

Its integral part $m = \left[\frac{363}{48}\right] = 7$

$\therefore$ T_{m+1} is the numerically greatest term in

$$\left(1+\frac{3x}{2}\right)^{10}$$

i.e. $T_{7+1} = T_8 = {}^{10}C_7\left(\frac{3x}{2}\right)^7$

$= {}^{10}C_7\left(\frac{3}{2}\times\frac{11}{8}\right)^7 = {}^{10}C_7\left(\frac{33}{16}\right)^7$

$\therefore$ N.G. term in the expansion of $(2 + 3x)^{10}$ is $= 2^{10}\cdot{}^{10}C_7\left(\frac{33}{16}\right)^7$.

ii) $(3x-4y)^{14}$ when x= 8, y = 3.

Sol. $(3x-4y)^{14} = \left(3x\left(1-\frac{4y}{3x}\right)\right)^{14}$

$$= (3x)^{14}\left(1-\frac{4y}{3x}\right)^{14}$$

Write $X = \frac{-4y}{3x} = -\left(\frac{4\times 3}{3\times 8}\right) = -\frac{1}{2}$

$|X| = \frac{1}{2}$

Now $\frac{(n+1)|X|}{1+|X|} = \frac{(14+1)\frac{1}{2}}{1+\frac{1}{2}} = 5$, an integer.

Here $|T_5| = |T_6|$ are N.G. terms.

T_5 in the expansion of $\left(1-\frac{4y}{3x}\right)^{14}$ is

$$T_5 = {}^{14}C_4\left(\frac{-4y}{3x}\right)^4 = {}^{14}C_4\left(\frac{1}{2}\right)^4$$

and $T_6 = {}^{14}C_5\left(\frac{-4y}{3x}\right)^5 = -{}^{14}C_5\left(\frac{1}{2}\right)^5$

Here N.G. terms are T_5 and T_6. They are

$$T_5 = {}^{14}C_4\left(\frac{1}{2}\right)^4 (24)^{14}$$

$$T_6 = -{}^{14}C_5\left(\frac{1}{2}\right)^5 (24)^{14}$$

But $|T_5| = |T_6|$

11. If R, n are positive integers, n is odd, 0 < F < 1 and if $(5\sqrt{5}+11)^n = R+F$, then prove that

i) R is an even integer and

ii) (R + F)F = 4^n.

Sol. i) Since R, n are positive integers, 0 < F < 1 and $(5\sqrt{5}+11)^n = R+F$

Let $(5\sqrt{5}-11)^n = f$

Now, $11 < 5\sqrt{5} < 12 \Rightarrow 0 < 5\sqrt{5}-11 < 1$

$\Rightarrow 0 < (5\sqrt{5}-11)^n < 1 \Rightarrow 0 < f < 1 \quad \Rightarrow 0 > -f > -1 \therefore -1 < -f < 0$

R + F – f = $(5\sqrt{5}+11)^n - (5\sqrt{5}-11)^n$

$$= \left[{}^nC_0(5\sqrt{5})^n + {}^nC_1(5\sqrt{5})^{n-1}(11) + {}^nC_2(5\sqrt{5})^{n-2}(11)^2 + \ldots + {}^nC_n(11)^n\right] - \left[{}^nC_0(5\sqrt{5})^n - {}^nC_1(5\sqrt{5})^{n-1}(11) + {}^nC_2(5\sqrt{5})^{n-2}(11)^2 + \ldots + {}^nC_n(-11)^n\right]$$

$$= 2\left[{}^nC_1(5\sqrt{5})^{n-1}(11) + {}^nC_3(5\sqrt{5})^{n-3}(11)^2 + \ldots\right]$$

= 2k where k is an integer.

∴ R + F – f is an even integer.

⇒ F – f is an integer since R is an integer.

But 0 < F < 1 and –1 < –f < 0 ⇒ –1 < F – f < 1

∴ F – f = 0 ⇒ F = f

∴ R is an even integer.

ii) (R + F)F = (R + F)f, $\because$ F = f

$= (5\sqrt{5}+11)^n(5\sqrt{5}-11)^n$

$= \left[(5\sqrt{5}+11)(5\sqrt{5}-11)\right]^n = (125-121)^n = 4^n$

∴ (R + F)F = 4^n.

12. If I, n are positive integers, 0 < f < 1 and if $(7+4\sqrt{3})^n = I+f$, then show that (i) I is an odd integer and (ii) (I + f)(I – f) = 1.

Sol. Given I, n are positive integers and

$(7+4\sqrt{3})^n = I+f$, 0 < f < 1

Let $7-4\sqrt{3} = F$

Now $6 < 4\sqrt{3} < 7 \Rightarrow -6 > -4\sqrt{3} > -7$

$\therefore$ Coefficient of x^8 in $\dfrac{(1+x)^2}{\left(1-\dfrac{2}{3}x\right)^3}$ is

$$= 45\left(\frac{2}{3}\right)^8 + 2\times 36\left(\frac{2}{3}\right)^7 + 28\left(\frac{2}{3}\right)^6$$

$$= \left(\frac{2}{3}\right)^6\left[45\times\frac{4}{9} + 72\times\frac{2}{3} + 28\right]$$

$$= \left(\frac{2}{3}\right)^6 (20+48+28) = \frac{96\times 2^6}{3^6} = \frac{2048}{243}$$

iii) Find the coefficient of x^7 in $\dfrac{(2+3x)^3}{(1-3x)^4}$.

Sol. $\dfrac{(2+3x)^3}{(1-3x)^4} = (2+3x)^3(1-3x)^{-4}$

$$= (8+36x+54x^2+27x^3)$$
$$[1 + {}^4C_1(3x) + {}^5C_2(3x)^2 + {}^6C_3(3x)^3 + {}^7C_4(3x)^4 + {}^8C_5(3x)^5 + {}^9C_6(3x)^6 + \ldots]$$

$\therefore$ Coefficient of x^7 in $\dfrac{(2+3x)^3}{(1-3x)^4}$ is

$$= 8\cdot\left({}^{10}C_7\cdot 3^7\right) + 36\cdot\left({}^9C_6(3)^6\right) + 54\left({}^8C_5(3^5)\right) + 27\left({}^7C_4(3^4)\right)$$

$$= 8\left({}^{10}C_3 3^7\right) + 36\left({}^9C_3 3^6\right) + 54\left({}^8C_3 3^5\right) + 27\left({}^7C_3 3^4\right)$$

18. Find the sum of the infinite series $\frac{7}{5}\left(1+\frac{1}{10^2}+\frac{1\cdot3}{1\cdot2}\frac{1}{10^4}+\frac{1\cdot3\cdot5}{1\cdot2\cdot3}\frac{1}{10^6}+...\right)$**.**

Sol. $1+\frac{1}{10^2}+\frac{1\cdot3}{1\cdot2}\frac{1}{10^4}+\frac{1\cdot3\cdot5}{1\cdot2\cdot3}\frac{1}{10^6}+...$

$$=1+\frac{1}{1!}\left(\frac{1}{100}\right)+\frac{1\cdot3}{2!}\left(\frac{1}{100}\right)^2+\frac{1\cdot3\cdot5}{3!}\left(\frac{1}{100}\right)^3+...$$

Comparing with $(1-x)^{-p/q}$

$$=1+\frac{p}{1!}\left(\frac{x}{q}\right)+\frac{p(p+q)}{2!}\left(\frac{x}{q}\right)^2 \quad p = 1, p+q=3, q= 2$$

$$\frac{x}{q}=\frac{1}{100}\Rightarrow x=\frac{q}{100}=\frac{2}{100}=0.02$$

$$\therefore 1+\frac{1}{10^2}+\frac{1\cdot3}{1\cdot2}\cdot\frac{1}{10^4}+...=(1-x)^{-p/q}$$

$$=(1-0.02)^{-1/2}=(0.98)^{-1/2}=\left(\frac{49}{50}\right)^{-1/2}=\left(\frac{50}{49}\right)^{1/2}=\frac{5\sqrt{2}}{7}$$

$$\therefore \frac{7}{5}\left[1+\frac{1}{10^2}+\frac{1\cdot3}{1\cdot2}\frac{1}{10^4}+\frac{1\cdot3\cdot5}{1\cdot2\cdot3}\frac{1}{10^6}+...\right]$$

$$=\frac{7}{5}\frac{5\sqrt{2}}{7}=\sqrt{2}$$

19. Show that

$$1+\frac{x}{2}+\frac{x(x-1)}{2\cdot4}+\frac{x(x-1)(x-2)}{2\cdot4\cdot6}+...$$
$$=1+\frac{x}{3}+\frac{x(x+1)}{3\cdot6}+\frac{x(x+1)(x+2)}{3\cdot6\cdot9}+...$$

Sol. L.H.S. = $1+\frac{x}{2}+\frac{x(x-1)}{2\cdot4}+\frac{x(x-1)(x-2)}{2\cdot4\cdot6}+...$

30. Find an approximate value of $\sqrt[6]{63}$ correct to 4 decimal places.

Sol. $\sqrt[6]{63} = (63)^{1/6} = (64-1)^{1/6}$

$$= (64)^{1/6}\left(1-\frac{1}{64}\right)^{1/6}$$

$$= 2\left[1-(0.5)^6\right]^{1/6}$$

$$= 2\left[\text{i} - \frac{\left(\frac{1}{6}\right)(0.5)^6}{1!} + \frac{\left(\frac{1}{6}\right)\left(\frac{1}{6}-1\right)}{2!}(0.5)^{12} + \ldots\right]$$

$$= 2[1-0.0026041] = 2[0.9973959]$$

$$= 1.9947918 = 1.9948 \text{ (correct to 4 decimals)}$$

31. If |x| is so small that x^2 and higher powers of x may be neglected, then find an approximate values of $\dfrac{\left(1+\frac{3x}{2}\right)^{-4}(8+9x)^{1/3}}{(1+2x)^2}$.

Sol. $\dfrac{\left(1+\frac{3x}{2}\right)^{-4}(8+9x)^{1/3}}{(1+2x)^2}$

$$= \left(1+\frac{3x}{2}\right)^{-4}\left[8\left(1+\frac{9}{8}x\right)\right]^{1/3}(1+2x)^{-2}$$

$$= \left(1+\frac{3x}{2}\right)^{-4}\cdot 8^{1/3}\left(1+\frac{9}{8}x\right)^{1/3}(1+2x)^{-2}$$

$$= 2\left[1-\frac{4}{1}\left(\frac{3x}{2}\right)\right]\left[1+\frac{1}{3}\left(\frac{9x}{8}\right)\right][1+(-2)(2x)]$$

$\because x^2$ and higher powers of x are neglecting

$$= 2(1-6x)\left(1+\frac{3x}{8}\right)(1-4x)$$

$$= 2\left(1-6x+\frac{3x}{8}\right)(1-4x)$$

33. Suppose that x and y are positive and x is very small when compared to y. Then find the approximate value of $\left(\frac{y}{y+x}\right)^{3/4}-\left(\frac{y}{y+x}\right)^{4/5}$**.**

Sol. $\left(\frac{y}{y+x}\right)^{3/4}-\left(\frac{y}{y+x}\right)^{4/5}$

$$=\left(\frac{y}{y\left(1+\frac{x}{y}\right)}\right)^{3/4}-\left(\frac{y}{y\left(1+\frac{x}{y}\right)}\right)^{4/5}$$

$$=\left(1+\frac{x}{y}\right)^{-3/4}-\left(1+\frac{x}{y}\right)^{-4/5}$$

$$=\left\{1+\left(\frac{-3}{4}\right)\left(\frac{x}{y}\right)+\frac{\left(-\frac{3}{4}\right)\left(\frac{-3}{4}-1\right)}{1\cdot 2}\left(\frac{x}{y}\right)^2+\ldots\right\}$$

$$-\left\{1+\left(\frac{-4}{5}\right)\left(\frac{x}{y}\right)+\frac{\left(\frac{-4}{5}\right)\left(\frac{-4}{5}-1\right)}{1\cdot 2}\left(\frac{x}{y}\right)^2+\ldots\right\}$$

(By neglecting $(x/y)^3$ and higher powers of x/y

$$=\left[1-\frac{3}{4}\left(\frac{x}{y}\right)-\frac{21}{32}\left(\frac{x}{y}\right)^2\right]-\left[1-\frac{4}{5}\left(\frac{x}{y}\right)+\frac{18}{25}\left(\frac{x}{y}\right)^2\right]$$

$$=\left(\frac{4}{5}-\frac{3}{4}\right)\frac{x}{y}-\left(\frac{21}{32}+\frac{18}{25}\right)\left(\frac{x}{y}\right)^2$$

$$=\frac{1}{20}\left(\frac{x}{y}\right)-\frac{1101}{800}\left(\frac{x}{y}\right)^2$$

$$\therefore \frac{4}{3}+2S=(1-x)^{-p/q}=\left(1-\frac{1}{2}\right)^{-2/3}$$

$$=\left(\frac{1}{2}\right)^{-2/3}=(2)^{2/3}=\sqrt[3]{4}$$

$$\therefore 2S=\sqrt[3]{4}-\frac{4}{3}\Rightarrow S=\frac{\sqrt[3]{4}}{2}-\frac{2}{3}=\frac{1}{\sqrt[3]{2}}-\frac{2}{3}$$

$$\therefore \frac{5}{6\cdot 12}+\frac{5\cdot 8}{6\cdot 12\cdot 18}+\frac{5\cdot 8\cdot 11}{6\cdot 12\cdot 18\cdot 24}+\ldots=\frac{1}{\sqrt[3]{2}}-\frac{2}{3}$$

36. If the coefficients of x^9, x^{10}, x^{11} in the expansion of $(1+x)^n$ are in A.P. then prove that $n^2-41n+398=0$.

Sol: Coefficient of x^r in the expansion $(1 - x)^n$ is ${}^nC_r$.

Given coefficients of x^9, x^{10}, x^{11} in the expansion of $(1 - x)^n$ are in A.P., then

$$2({}^nC_{10})={}^nC_9+{}^nC_{11}$$

$$\Rightarrow 2\frac{n!}{(n-10)!10!}=\frac{n!}{(n-9)!9!}+\frac{n!}{(n-11)!11!}$$

$$\Rightarrow \frac{2}{10(n-10)}=\frac{1}{(n-9)(n-10)}+\frac{1}{11\times 10}$$

$$\Rightarrow \frac{2}{(n-10)10}=\frac{110+(n-9)(n-10)}{110(n-9)(n-10)}$$

$$\Rightarrow 22(n-9)=110+n^2-19n+90$$

$$\Rightarrow n^2-41n+398=0$$