Compensator can be electrical, mechanical, pneumatic or hydrolic type of device. Mostly electrical networks are used as compensator in most of the control system. The very simplest of these are Lead, lag & lead-lag networks.

Lead Compensator

Lead compensator are used to improve the transient response of a system.

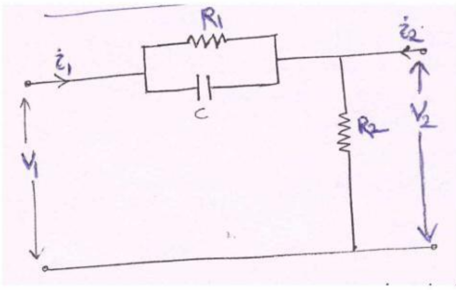

Fig: Electric Lead Network

Taking i_2=0 & applying Laplace Transform, we get

$$\frac{V_2(s)}{V_1(s)} = \frac{R_2(R_1Cs + 1)}{R_2 + R_2R_1Cs + R_1}$$

Let $\tau = R_1C$, $\alpha = \frac{R2}{R_1+R_2} < 1$

$\frac{V_2(s)}{V_1(s)} = \frac{\alpha(\tau s+1)}{(1+\tau\alpha s)}$ Transfer function of Lead Compensator

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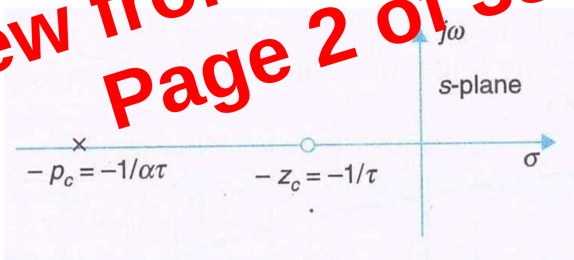

Fig: S-Plane representation of Lead Compensator

Bode plot for Lead Compensator

Maximum phase lead occurs at $w_m = \frac{1}{\tau\sqrt{\alpha}}$

Let ϕ_m = maximum phase lead

$$\sin\phi_m = \frac{1-\alpha}{1+\alpha}$$

$$\alpha = \frac{1-\sin\phi_m}{1+\sin\phi_m}$$

Magnitude at maximum phase lead $G_c(jw) = \frac{1}{\sqrt{\alpha}}$

based on the assumption the system would be dominated by these two complex pole therefore its dynamic behavior can be approximated by that of a second order system.

A compensator is now designed so that the least damped complex pole of the resulting transfer function correspond to the desired dominant pole & all other closed loop poles are located very close to the open loop zeros or relatively far away from the jw axis. This ensures that the poles other than the dominant poles make negligible contribution to the system dynamics.

Lead Compensation

- Consider a unity feedback system with a forward path unalterable Transfer function $G_f(s)$, then let the dynamic response specifications are translated into desired location S_d for the dominant complex closed loop poles.
- If the angle criteria as S_d is not meet i.e $\angle G_f(s) \neq \pm 180°$ the uncompensated Root Locus with variable open loop gain will not pass through the desired root location, indicating the need for the compensation.
- The lead compensator $G_c(s)$ has to be designed that the compensated root locus passes through S_d . In terms of angle criteria this requires that

$$\angle G_c\ s_d\ G_f\ s_d = \angle G_c\ s_d + \angle G_f\ s_d \pm 180°$$

$$\angle G_c\ s_d = \phi = \pm 180° - \angle G_f\ s_d$$

- Thus for the root locus for the compensated system to pass through the desired root location the lead compensator pole-zero pair must contribute an angle ϕ .
- For a given angle ϕ required for lead compensation there is no unique location for pole-zero pair. The best compensator pole-zero location is the one which gives the largest value of α.

Where $\alpha = \frac{z_c}{p_c}$

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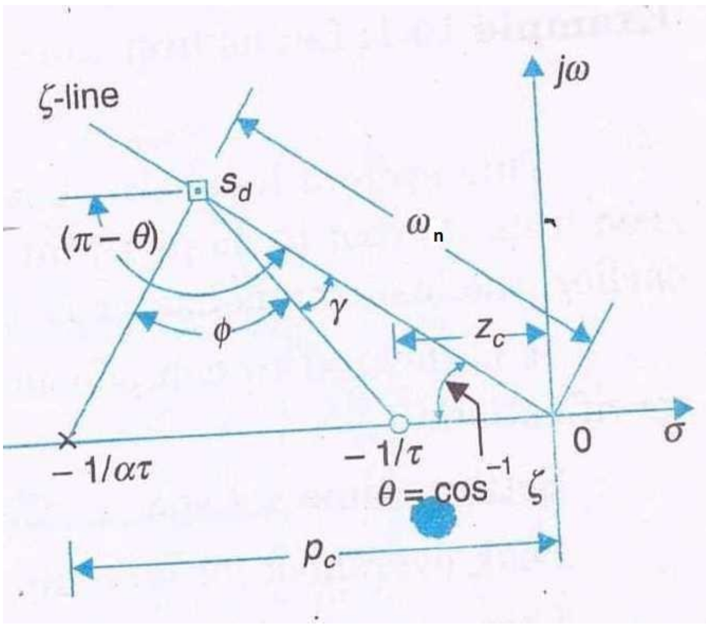

Fig: Angle contribution of Lead compensator

Step5: Find the frequency ω_m at which the uncompensated system will have a gain equals to $-10\log\frac{1}{\alpha}$ from the bode plot drawn.

Take $\omega_{c2} = \omega_m$ = cross-over frequency of compensated system.

Step6: Corner frequency of the network are calculated as

$\omega_1 = \frac{1}{\tau} = \omega_m\sqrt{\alpha}$, $\omega_2 = \frac{1}{\tau\alpha} = \frac{\omega_m}{\sqrt{\alpha}}$

Transfer function for compensated system in Lead network $G_c(s) = \frac{s+\frac{1}{\tau}}{s+\frac{1}{\tau\alpha}}$

Step7: Draw the magnitude & Phase plot for the compensated system & check the resulting phase margin. If the phase margin is still low raise the value of ϵ & repeat the procedure.

Lag Compensation

Procedure of Lead Compensation

Step1: Determine the value of loop gain K to satisfy the specified error constant.

Step2: For this value of K draw the bode plot & determine the phase margin ϕ for the system.

Step3: If ϕ_s =specified phase margin &

ϕ = phase margin of uncompensated system (found out from the bode plot drawn)

ϵ =margin of safety $(5° - 10°)$

- For a suitable ϵ find $\phi_2 = \phi_s + \epsilon$,where ϕ_2 is measured above $-180°$ line.

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Step4: Find the frequency ω_{c2} where the uncompensated system makes a phase margin contribution of ϕ_2.

Step5: Measure the gain of uncompensated system at ω_{c2}. Find β from the equation

$$gain\ at\ \omega_{c2} = 20\log\beta$$

Step6: Choose the upper corner frequency $\omega_2 = \frac{1}{\tau}$ of the network one octave to one decade below ω_{c2} $(i.e\ between\ \frac{\omega_{c2}}{2}\ to\ \frac{\omega_{c2}}{10})$

Step7: Thus β & τ are determined which can be used to find the transfer function of Lag compensator.

$$G_c(s) = \frac{1}{\beta}\,\frac{s+\frac{1}{\tau}}{s+\frac{1}{\tau\alpha}}$$

Compensated Transfer function $G(s) = G_f\,G_c$

Draw the bode plot of the compensated system & check if the given specification are met.

MATLab Code

Plotting rootlocus with MATLAB(**rlocus**)

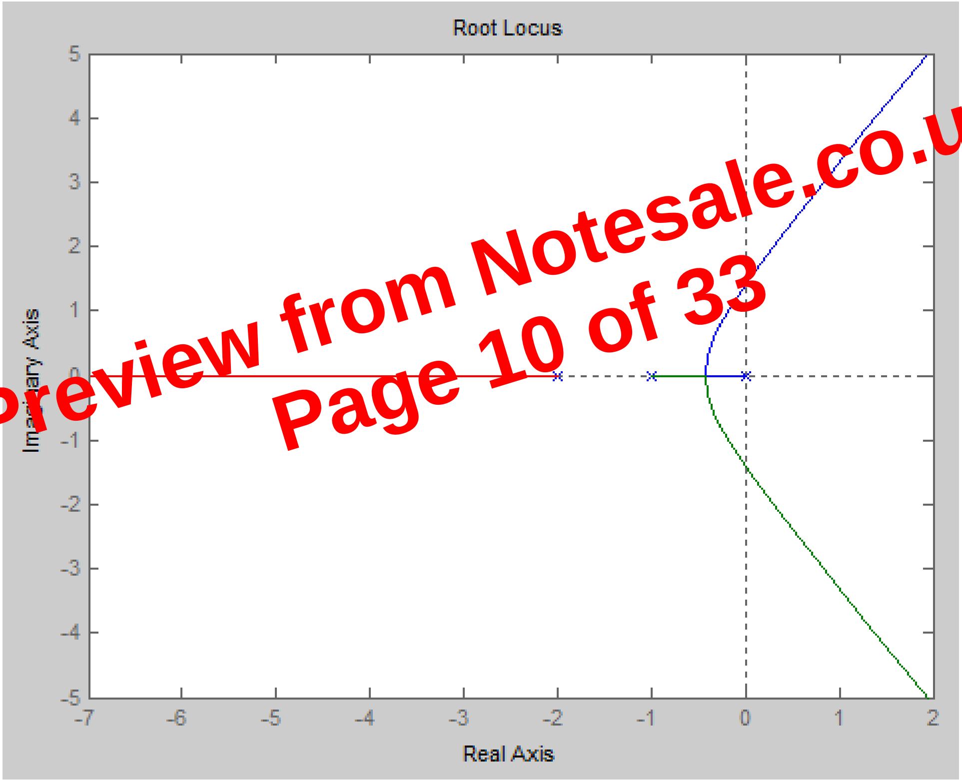
Consider a unity-feedback control system with the following feedforward transfer function:

$$G(s) = \frac{K}{s(s+1)(s+2)}$$

Using MATLAB, plot the rootlocus.

$$G(s) = \frac{K}{s(s+1)(s+2)} = \frac{K}{s^3 + 3s^2 + 2s}$$

```
num=[1];
den=[1 3 2 0];
h = tf(num,den);
rlocus(h)
```



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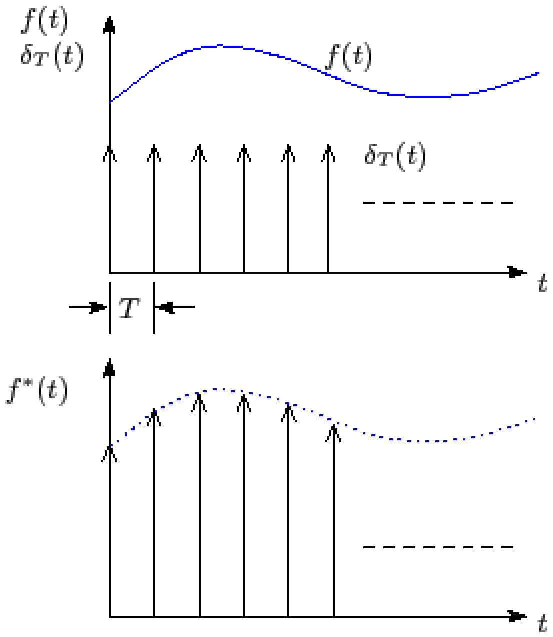
Plotting Bode Diagram with MATLAB(**bode**)

Consider the following transfer function

$$G(s) = \frac{25}{s^2 + 4s + 25}$$

Plot the Bode diagram for this transfer function



the output of an ideal sampler can be expressed as

$$f^*(t) = \sum_{k=0}^{\infty} f(kT)\delta(t-kT)$$

$$\Rightarrow F^*(s) = \sum_{k=0}^{\infty} f(kT)e^{-kTs}$$

One should remember that practically the output of a sampler is always followed by a hold device which is the reason behind the name sample and hold device. Now, the output of a hold device will be the same regardless the nature of the sampler and the attenuation factor p can be dropped in that case. Thus the sampling process can be be always approximated by an ideal sampler or impulse modulator.

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Z- Transform

Let the output of an ideal sampler be denoted by $f^*(t)$

$$L\ f^*(t) = f^*(s) = \sum_{K=0}^{\infty} f(KT)\, e^{-KTs}$$

If we substitute $Z = e^{Ts}$,then we get $F(z)$, is the Z-transform of $f(t)$ at the sampling instants k

$$F(z) = \sum_{k=0}^{\infty} f(kT)z^{-k}$$