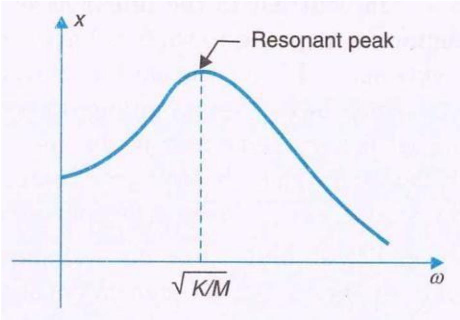


Fig. 3.2 Frequency response curve of spring-mass-damper system

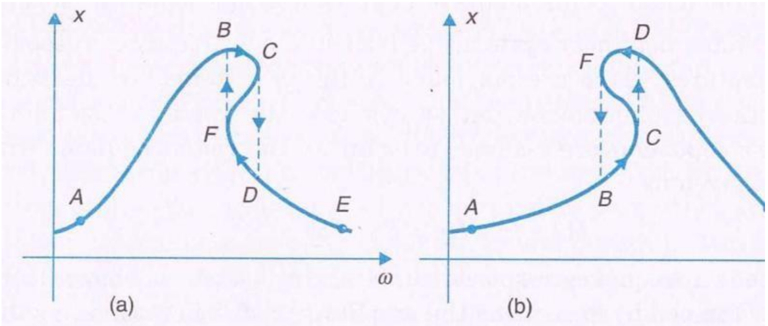


Fig. 3.3 (a) Jump resonance in nonlinear system(hard spring case);
(b) Jump resonance in nonlinear system(hard spring case).

Let us now assume that the restoring force of the spring is nonlinear,given by $K_1x + K_2x^3$.The nonlinear spring characteristic is shown in Fig3.1(b). Now the system equation becomes

$$Mx + fx + K_1x + K_2x^3 = F\cos\omega t \ldots\ldots\ldots\ldots (3.2)$$

The frequency response curve for the hard spring($K_2 > 0$) is shown in Fig3.3(a).

For a hard spring, as the input frequency is gradually increased from zero, the measured two response follows the curve through the A, B and C, but at C an increment in frequency results in discontinuous jump down to the point D, after which with further increase in frequency, the response curve follows through DE. If the frequency is now decreased, the response follows the curve EDF with a jump up to B from the point F and then the response curve moves towards A. This phenomenon which is peculiar to nonlinear systems is known as jump resonance. For a soft spring, jump phenomenon will happen as shown in fig. 3.3(b).

<u>Methods of Analysis</u>

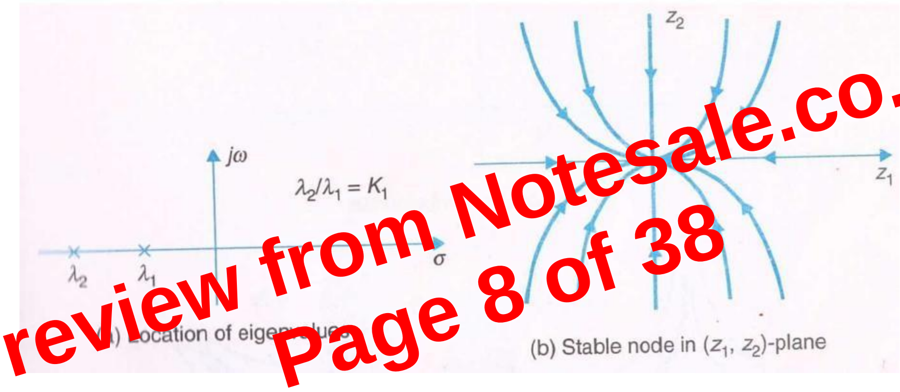
Nonlinear systems are difficult to analyse and arriving at general conclusions are tedious. However, starting with the classical techniques for the solution of standard nonlinear differential equations, several techniques have been evolved which suit different types of analysis. It should be emphasised that very often the conclusions arrived at will be useful for the system under specified conditions and do not always lead to generalisations. The commonly used methods are listed below.

regions. Since the solutions from each of the initial conditions are unique, the phase trajectories do not cross one another. If the system has nonlinear elements which are piece-wise linear, the complete state space can be divided into different regions and phase plane trajectories constructed for each of the regions separately.

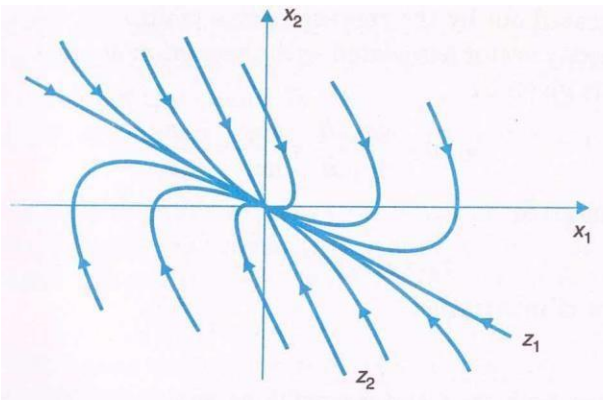
<u>Analysis & Classification of Singular Points</u>

Nodal Point: Consider eigen values are real, distinct and negative as shown in figure3.9 (a). For this case the equation of the phase trajectory follows as $z_2 = c\, z_1^{\lambda_2/\lambda_1}$ Where c is an integration constant . The trajectories become a set of parabola as shown in figure 3.9(b) and the equilibrium point is called a node. In the original system of coordinates, these trajectories appear to be skewed as shown in figure 3.9(c).

If the eigen values are both positive, the nature of the trajectories does not change, except that the trajectories diverge out from the equilibrium point as both $z_1(t)$ and $z_2(t)$ are increasing exponentially. The phase trajectories in the x_1-x_2 plane are as shown in figure3.9 (d). This type of singularity is identified as a node, but it is an unstable node as the trajectories diverge from the equilibrium point.



(a) Location of eigenvalues
(b) Stable node in (z_1, z_2)-plane



(c) Stable node in (X_1,X_2)-plane

Instability

It may be noted that instability in a nonlinear system can be established by direct recourse to the instability theorem of the direct method .The basic instability theorem is presented below:

Theorem-4
Consider a system

$$\dot{x} = f(x);\ f(0) = 0$$

Suppose there a exist a scalar function W(x) which, for real number $\in > 0$, satisfies the following properties for all x in the region $\|X\| < \epsilon$;

(a) W(x)>0; $x \neq 0$
(b) W (0) = 0
(c) W(x) has continuous partial derivatives with respect to all component of x
(d) $\frac{dW}{dt} \geq 0$
Then the system is unstable at the origin.

Direct Method of Liapunov & the Linear System:

In case of linear systems, the direct method of liapunov provides a simple approach to stability analysis. It must be emphasized that compared to the results presented, no new results are obtained by the use of direct method for the stability analysis of linear systems. However, the study of linear systems using the direct method is quite useful because it extends our thinking to nonlinear systems.
Consider a linear autonomous system described by the state equation

$$\dot{X} = AX \text{.......................................} (3.6)$$

The linear system is asymptotically stable in-the-large at the origin if and only if given any symmetric, positive definite matrix **Q,** there exists a symmetric positive definite matrix **P** which is the unique solution

$$A^{T}P + PA = -Q \text{..........................} (3.7)$$

Proof
To prove the sufficiency of the result of above theorem, let us assume that a symmetric positive definite matrix **P** exists which is the unique solution of eqn.(3.8). Consider thescalar function.

$$V(\mathbf{x}) = \mathbf{x}^{T}\mathbf{P}\mathbf{x}$$

Note that $V(\mathbf{x}) > 0$ for $\mathbf{x} \neq \mathbf{0}$

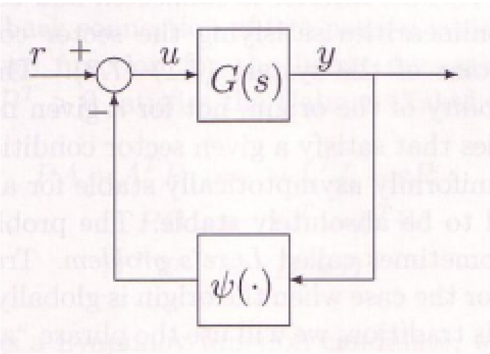
$$V(\mathbf{0}) = \mathbf{0}$$

And

The time derivate of V(x) is

POPOV CRITERION

Many nonlinear physical systems can be represented as a feedback connection of a linear dynamical system and a nonlinear element.



The process for representing a system in this form depends on the particular system involved. For instance, in the case in which a control system's only nonlinearity is in the form of a relay or actuator/sensor nonlinearity, there is no difficulty in representing the system in this feedback form. In other cases, the representation may be less obvious. We assume that the extern input $r = 0$ and study the behavior of the unforced system. What is unique about this chapter is the use of the frequency response of the linear system, which builds on classical control tools like Nyquist plot and Nyquist criterion.
The system is said to be absolutely stable if it has a globally uniformly asymptotically stable equilibrium point at the origin for all nonlinearities in a given sector. The circle and Popov criteria give frequency-domain sufficient conditions for absolute stability in the form of strict positive realness of certain transfer functions. In the single-input-single-output case, both criteria can be applied graphically.

We assume the external input $r = 0$ and study the behavior of the unforced system represented by

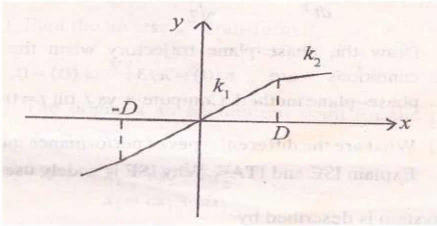
$$\dot{x} = Ax + Bu$$
$$y = Cx + Du$$
$$u = -\psi(y)$$

where $x \in \Re^n, u, y \in \Re^p, (A,B)$ is controllable, (A,C) is observable, and $\psi : [0,\infty) \times \Re^p \to \Re^p$ is a memoryless, possibly time-varying, nonlinearity, which is piecewise continuous in t and locally Lipschitz in y.

Definition 1.1: A memoryless function $h : [0,\infty) \to \Re$ is said to be to belong to the sector

1) $[0,\infty]$ if $u \cdot h(t,u) \geq 0$

12. Derive the expression for describing function of the following non-linearity as shown in figure below. **[14]**



13. Describe Lyaponov‟s stability criterion. **[3]**

14. What do you mean by sign definiteness of a function? Check the positive definiteness of

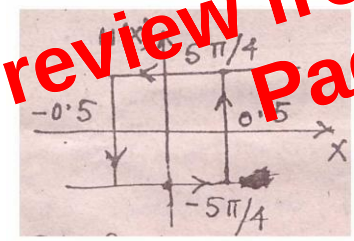
$$V(X) = x_1^2 + \frac{2x_2^2}{1+x_2^2}$$ **[4]**

15. Distinguish between the concepts of stability, asymptotic stability & global stability. **[4]**

16. (a) What are singular points in a phase plane? Explain the following types of singularity with sketches: **[9]**

Stable node, unstable node, saddle point, stable focus, unstable focus, vortex.

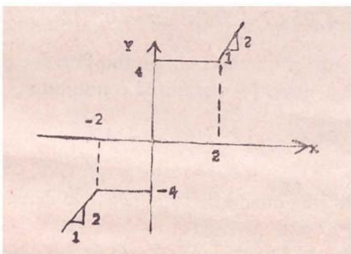
(b) Obtain the describing function of N(x) in figure below. Derive the formula used.



[6]

17. (a) Evaluate the describing function of the non linear element shown in figure below.

[6]



24. (a) A non-linear system is governed by

$$\frac{d^2x}{dt^2} + 8x - 4x^2 = 0$$

Determine the singular point s and their nature. Plot the trajectory passing through((X_1 = 2, X_2 = 0) without any approximation.

(b) What are the limitations of phase-plane analysis. **[12+3]**

25.(a) Find the describing function of the following type of non linearities. **[8]**

i)ideal on off relay

ii)ideal saturation

(b) Derive a Liapunov function for the defined by **[8]**

$$x_1 = x_2$$

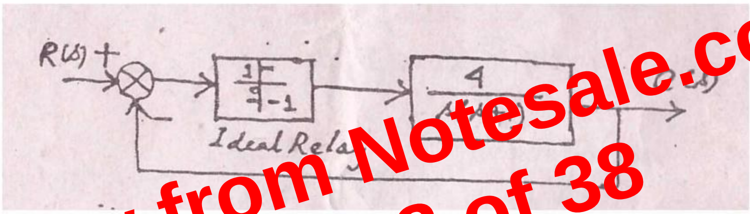
$$x_2 = -3x_1{}^2 - 3\,x_2$$

Also check the stability of the system.

26.(a) Determine the singular points in the phase plane and sketch the plane trajectories for a system of characteristics equation

$$\frac{d^2x(t)}{dt^2} + 8x\ t\ \ - 4x^2\ t\ \ = 0$$ **[8]**

(b) A system described by the system shown in fig below



Will there be a limit cycle? If so determine its amplitude and frequency. **[8]**