MODULE-IV

OPTIMAL CONTROL SYSTEMS

Introduction:

There are two approaches to the design of control systems. In one approach we select the configuration of the overall system by introducing compensators to meet the given specifications on the performance. In other approach, for a given plant we find an overall system that meets the given specifications & then compute the necessary compensators.

The classical design based on the first approach, the designer is given a set of specifications in time domain or in frequency domain & system configuration. Compensators are selected that give as closely as possible, the desired system performance. In general, it may not be possible to satisfy all the desired specifications. Then, through a trial & error procedure, an acceptable system performance is achieved.

The trial & error uncertainties are eliminated in the parameter optimization method. In parameter optimization procedure, the performance specification consists of a single performance index. For a fixed system configuration, parameters that minimize the performance index are selected.

Parameter Optimization: Servomechanisms

The analytical approach of parameter optimization consists of the following steps:-

(i) Compute the performance index J as a function of the free parameters $K_1, K_2, \ldots, K_n$ of the system with fixed configuration:

$J=f(K_1,K_2,\ldots,K_n)$............................(1)

(ii) Determine the solution set K_i of the equations

$$\frac{\partial J}{\partial K_i}=0; \quad i=1,2,\ldots\ldots n \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

Equation (2) give the necessary conditions for J to be minimum.

Sufficient conditions

From the solution set of equation(2), find the subset that satisfies the sufficient conditions which require that the Hessian matrix given below is positive definite.

$$H=\begin{bmatrix} \frac{\partial^2 J}{\partial K_1^2} & \frac{\partial^2 J}{\partial k_1 \partial k_2} \cdots\cdots & \frac{\partial^2 J}{\partial k_1 \partial k_n} \\ \frac{\partial^2 J}{\partial k_2 \partial k_1} & \frac{\partial^2 J}{\partial K_2^2} \cdots\cdots & \frac{\partial^2 J}{\partial k_2 \partial k_n} \\ \cdots\cdots & \cdots\cdots & \cdots\cdots \\ \frac{\partial^2 J}{\partial k_n \partial k_1} & \frac{\partial^2 J}{\partial k_n \partial k_2} \cdots\cdots & \frac{\partial^2 J}{\partial K_n^2} \end{bmatrix} \qquad \ldots\ldots\ldots\ldots(3)$$

$$u^*(t) = -R^{-1}B^TPX(t) = -\frac{1}{2}\begin{bmatrix}0 & 1\end{bmatrix}\begin{bmatrix}2\sqrt{2} & 2 \\ 2 & 2\sqrt{2}\end{bmatrix}\begin{bmatrix}x_1(t) \\ x_2(t)\end{bmatrix} = -x_1(t) - \sqrt{2}\,x_2(t)$$

It can be easily verified that closed loop system is asymptotically stable.(Though Q is positive definite)

The Output Regulator Problem

In the state regulator problem, we are concerned with making all the components of the state vector X(t) small.In the output regulator problem on the other hand, we ere concerned with making the components of the output vector small.

Consider an observable controlled process described by the equations

$$\dot{X}(t) = AX(t) + Bu(t) \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad 17$$

$$Y(t) = CX(t)$$

Find the optimal control law u*(t),$t \in [t_0, t_f]$, where t_0 & t_f are specified initial & final times respectively, so that the optimal PI

$$J = \frac{1}{2}Y^T(t_f)\,F\,Y(t_f) + \frac{1}{2}\int_{t_o}^{t_f}\left[Y^T(t)\,QY(t) + u^T(t)\,R\,u(t)\right]dt \ldots\ldots\ldots\ldots\ldots\ldots (18)$$

Is minimized,subject to initial state $x(t_0)=x_0$.

Tracking Problem

Consider an observable controlled process described by the equation(17). Suppose that the vector Z(t) is the desired output.

The error vector e(t)=Z(t)-Y(t)

Find the optimal control law u*(t),$t \in [t_0, t_f]$, where t_0 & t_f are specified initial & final times respectively, so that the optimal PI

$$J = \frac{1}{2}e^T(t_f)\,F\,e(t_f) + \frac{1}{2}\int_{t_o}^{t_f}\left[e^T(t)\,Qe(t) + u^T(t)\,R\,u(t)\right]dt$$

Is minimized .

Output Regulator as state regulator Problem

If the controlled process given by equation(17) is observable then, we can reduce the output regulator problem to the state regulator problem.

(ii) Find the minimum value of J

(iii) Find sensitivity of J with respect to k **[15]**

8. A linear autonomous system is described in the state equation

$$\dot{X} = \begin{bmatrix} -4K & 4K \\ 2K & -6K \end{bmatrix} X$$

Find restriction on the parameter k to guarantee stability of the system. **[15]**

9. A first order system is described by the differential equation

$$\dot{X}(t) = 2X(t) + u(t)$$

Find the control law that minimises the performance index

$$J = \frac{1}{2}\int_0^{t_f} (3X^2 + \frac{1}{4}u^2)\, dt$$

When t_f =1 second **[15]**

10.(a) What do you understand about parameter optimisation of regulator? **[6]**

(b) Find the control laws which minimises the performance index **[10]**

$$J = \int_0^{\infty} (x_1^2 + u^2)\, dt$$

For the system

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$