41 degrees and 29 minutes is written as 41°29′. 41°29′ is equivalent to $41\frac{29°}{60} = 41.483°$ as a decimal, correct to 3 decimal places by calculator.
1 minute further subdivides into 60 seconds,

i.e. **1 minute = 60 seconds**

which is written as **1′ = 60″**.

(Notice that for minutes, 1 dash is used and for seconds, 2 dashes are used.)
For example, 56 degrees, 36 minutes and 13 seconds is written as 56°36′13″.

20.2.2 Radians and degrees

One radian is defined as the angle subtended at the centre of a circle by an arc equal in length to the radius. (For more on circles, see Chapter 26.)
With reference to Figure 20.2, for arc length s, θ radians $= \frac{s}{r}$

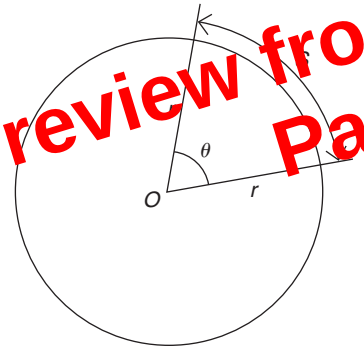


Figure 20.2

When s is the whole circumference, i.e. when $s = 2\pi r$,

$$\theta = \frac{s}{r} = \frac{2\pi r}{r} = 2\pi$$

In one revolution, $\theta = 360°$. Hence, the relationship between **degrees and radians** is

$$360° = 2\pi \text{ radians or } \mathbf{180° = \pi\ rad}$$

i.e. $$\mathbf{1\ rad = \frac{180°}{\pi} \approx 57.30°}$$

Here are some worked examples on angular measurement.

Problem 1. Evaluate 43°29′ + 27°43′

$$\begin{array}{r} 43°\ 29' \\ +\ 27°\ 43' \\ \hline 71°\ 12' \\ 1° \end{array}$$

(i) 29′ + 43′ = 72′

(ii) Since 60′ = 1°, 72′ = 1°12′

(iii) The 12′ is placed in the minutes column and 1° is carried in the degrees column.

(iv) 43° + 27° + 1° (carried) = 71°. Place 71° in the degrees column.

This answer can be obtained using the **calculator** as follows.

1. Enter 43
2. Press ° ′ ″
3. Enter 29
4. Press ° ′ ″
5. Press +
6. Enter 27
7. Press ° ′ ″
8. Enter 43
9. Press ° ′ ″
10. Press =

Answer = **71°12′**

Thus, **43°29′ + 27°43′ = 71°12′**.

Problem 2. Evaluate 84°13′ − 56°39′

$$\begin{array}{r} 84°\ 13' \\ -\ 56°\ 39' \\ \hline 27°\ 34' \end{array}$$

(i) 13′ − 39′ cannot be done.

(ii) 1° or 60′ is 'borrowed' from the degrees column, which leaves 83° in that column.

(iii) (60′ + 13′) − 39′ = 34′, which is placed in the minutes column.

(iv) 83° − 56° = 27°, which is placed in the degrees column.

This answer can be obtained using the **calculator** as follows.

1. Enter 84
2. Press ° ′ ″
3. Enter 13
4. Press ° ′ ″
5. Press −
6. Enter 56
7. Press ° ′ ″
8. Enter 39
9. Press ° ′ ″
10. Press =

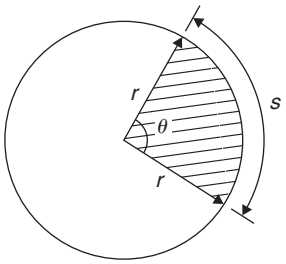
Answer = **27°34′**

Thus, **84°13′ − 56°39′ = 27°34′**.

Problem 3. Evaluate 19° 51′47″ + 63°27′34″

$$\begin{array}{r} 19°\ 51'\ 47'' \\ +\ 63°\ 27'\ 34'' \\ \hline 83°\ 19'\ 21'' \\ 1°\ \ 1' \end{array}$$

(v) **Circle** Area $= \pi r^2$ Circumference $= 2\pi r$



Radian measure: 2π radians $= 360$ degrees

For a sector of circle:

arc length, $s = \dfrac{\theta^\circ}{360}(2\pi r) = r\theta$ (θ in rad)

shaded area $= \dfrac{\theta^\circ}{360}(\pi r^2) = \dfrac{1}{2}r^2\theta$ (θ in rad)

Equation of a circle, centre at origin, radius r:

$$x^2 + y^2 = r^2$$

Equation of a circle, centre at (a, b), radius r:

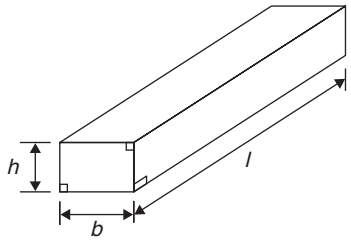
$$(x-a)^2 + (y-b)^2 = r^2$$

Volumes and surface areas of regular solids:

(i) **Rectangular prism (or cuboid)**

Volume $= l \times b \times h$

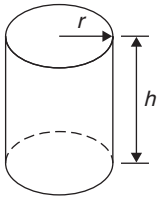
Surface area $= 2(bh + hl + lb)$



(ii) **Cylinder**

Volume $= \pi r^2 h$

Total surface area $= 2\pi rh + 2\pi r^2$

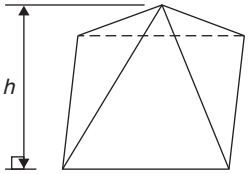


(iii) **Pyramid**

If area of base $= A$ and

perpendicular height $= h$ then:

$$\text{Volume} = \frac{1}{3} \times A \times h$$



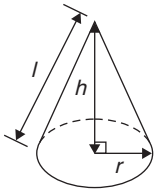
Total surface area = sum of areas of triangles forming sides + area of base

(iv) **Cone**

Volume $= \dfrac{1}{3}\pi r^2 h$

Curved Surface area $= \pi rl$

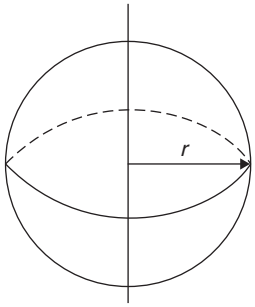
Total Surface area $= \pi rl + \pi r^2$



(v) **Sphere**

$$\text{Volume} = \frac{4}{3}\pi r^3$$

Surface area $= 4\pi r^2$



Chapter 10

Exercise 39 (page 69)

1. x^2+5x+6 **2.** $2x^2+9x+4$
3. $4x^2+12x+9$ **4.** $2j^2+2j-12$
5. $4x^2+22x+30$ **6.** $2pqr+p^2q^2+r^2$
7. $a^2+2ab+b^2$ **8.** $x^2+12x+36$
9. $a^2-2ac+c^2$ **10.** $25x^2+30x+9$
11. $4x^2-24x+36$ **12.** $4x^2-9$
13. $64x^2+64x+16$ **14.** $r^2s^2+2rst+t^2$
15. $3ab-6a^2$ **16.** $2x^2-2xy$
17. $2a^2-3ab-5b^2$ **18.** $13p-7q$
19. $7x-y-4z$ **20.** $4a^2-25b^2$
21. $x^2-4xy+4y^2$ **22.** $9a^2-6ab+b^2$
23. 0 **24.** $4-a$
25. $4ab-8a^2$ **26.** $3xy+9x^2y-15x^2$
27. $2+5b^2$ **28.** $11q-2p$

Exercise 40 (page 71)

1. $2(x+2)$ **2.** $2x(y-4)$
3. $p(b+2c)$ **4.** $7x(1+2y)$
5. $4d(d-3f^5)$ **6.** $4x(1+2x)$
7. $2y(6-4y)$ **8.** $x(1+3x-5x^2)$
9. $ba(a+b^2)$ **10.** $-(s+p+t)$
11. $3xy(xy^3-5y+6)$ **12.** $2pq^2(2p^2-5q)$
13. $7ab(3ab-4)$ **14.** $2xy(y+3x+4x^2)$
15. $2xy\left(x-2y^2+4x^2y^3\right)$ **16.** $7y(4+y+2x)$
17. $\dfrac{3x}{y}$ **18.** 0 **19.** $\dfrac{2r}{t}$
20. $(a+b)(y+1)$ **21.** $(p+q)(x+y)$
22. $(x-y)(a+b)$ **23.** $(a-2b)(2x+3y)$

Exercise 41 (page 72)

1. $2x+8x^2$ **2.** $12y^2-3y$
3. $4b-15b^2$ **4.** $4+3a$
5. $\dfrac{3}{2}-4x$ **6.** 1
7. $10y^2-3y+\dfrac{1}{4}$ **8.** $9x^2+\dfrac{1}{3}-4x$
9. $6a^2+5a-\dfrac{1}{7}$ **10.** $-15t$
11. $\dfrac{1}{5}-x-x^2$ **12.** $10a^2-3a+2$

Chapter 11

Exercise 42 (page 75)

1. 1 **2.** 2 **3.** 6 **4.** -4 **5.** 2
6. 1 **7.** 2 **8.** $\dfrac{1}{2}$ **9.** 0 **10.** 3
11. 2 **12.** -10 **13.** 6 **14.** -2 **15.** 2.5
16. 2 **17.** 6 **18.** -3

Exercise 43 (page 76)

1. 5 **2.** -2 **3.** $-4\dfrac{1}{2}$ **4.** 2 **5.** 12
6. 15 **7.** -4 **8.** $5\dfrac{1}{3}$ **9.** 2 **10.** 13
11. -10 **12.** 2 **13.** 3 **14.** -11 **15.** -6
16. 9 **17.** $6\dfrac{1}{4}$ **18.** [illegible] **19.** 4 **20.** 10
21. 2 **22.** $-3\dfrac{1}{3}$ **23.** ± 3 **24.** ± 4

Exercise 44 (page 79)

1. 10^{-7} **2.** $8\,\text{m/s}^2$ **3.** 3.472
4. (a) $1.8\,\Omega$ (b) $30\,\Omega$
5. digital camera battery £9, camcorder battery £14
6. $800\,\Omega$ **7.** $30\,\text{m/s}^2$

Exercise 45 (page 80)

1. 12 cm, 240 cm^2 **2.** 0.004 **3.** 30
4. 45°C **5.** 50 **6.** £312, £240
7. 30 kg **8.** 12 m, 8 m **9.** 3.5 N

Chapter 12

Exercise 46 (page 84)

1. $d=c-e-a-b$ **2.** $x=\dfrac{y}{7}$
3. $v=\dfrac{c}{p}$ **4.** $a=\dfrac{v-u}{t}$
5. $R=\dfrac{V}{I}$ **6.** $y=\dfrac{1}{3}(t-x)$
7. $r=\dfrac{c}{2\pi}$ **8.** $x=\dfrac{y-c}{m}$

9. (a) $\frac{1}{5}$ (b) 6 (c) $E = \frac{1}{5}L + 6$ (d) 12 N (e) 65 N

10. $a = 0.85, b = 12$, 254.3 kPa, 275.5 kPa, 280 K

Chapter 18

Exercise 70 (page 149)

1. (a) y (b) x^2 (c) c (d) d **2.** (a) y (b) $\sqrt{x}$ (c) b (d) a

3. (a) y (b) $\frac{1}{x}$ (c) f (d) e **4.** (a) $\frac{y}{x}$ (b) x (c) b (d) c

5. (a) $\frac{y}{x}$ (b) $\frac{1}{x^2}$ (c) a (d) b

6. $a = 1.5, b = 0.4$, 11.78 mm^2 **7.** $y = 2x^2 + 7$, 5.15

8. (a) 950 (b) 317 kN

9. $a = 0.4, b = 8.6$ (i) 94.4 (ii) 11.2

Exercise 71 (page 154)

1. (a) lg y (b) x (c) lg a (d) lg b

2. (a) lg y (b) lg x (c) L (d) lg k

3. (a) ln y (b) x (c) n (d) ln m

4. $I = 0.0012\,V^2$, 6.75 candelas

5. $a = 3.0, b = 0.5$

6. $a = 5.7, b = 2.6$, 38.53, 3.0

7. $R_0 = 26.0, c = 1.42$ **8.** $y = 0.08e^{0.24x}$

9. $T_0 = 35.4$ N, $\mu = 0.27$, 65.0 N, 1.28 radians

Chapter 19

Exercise 72 (page 156)

1. $x = 2, y = 4$ **2.** $x = 1, y = 1$

3. $x = 3.5, y = 1.5$ **4.** $x = -1, y = 2$

5. $x = 2.3, y = -1.2$ **6.** $x = -2, y = -3$

7. $a = 0.4, b = 1.6$

Exercise 73 (page 160)

1. (a) Minimum (0, 0) (b) Minimum (0, −1)
(c) Maximum (0, 3) (d) Maximum (0, −1)

2. −0.4 or 0.6 **3.** −3.9 or 6.9

4. −1.1 or 4.1 **5.** −1.8 or 2.2

6. $x = -1.5$ or −2, Minimum at (−1.75, −0.1)

7. $x = -0.7$ or 1.6 **8.** (a) ± 1.63 (b) 1 or −0.3

9. (−2.6, 13.2), (0.6, 0.8); $x = -2.6$ or 0.6

10. $x = -1.2$ or 2.5 (a) −30 (b) 2.75 and −1.50
(c) 2.3 or −0.8

Exercise 74 (page 161)

1. $x = 4, y = 8$ and $x = -0.5, y = -5.5$

2. (a) $x = -1.5$ or 3.5 (b) $x = -1.24$ or 3.24
(c) $x = -1.5$ or 3.0

Exercise 75 (page 162)

1. $x = -2.0, -0.5$ or 1.5

2. $x = -2$, 1 or 3, Minimum at (2.1, −4.1),
Maximum at (−0.8, 8.2)

3. $x = 1$ **4.** $x = -2.0$, 0.4 or 2.6

5. $x = 0.7$ or 2.5

6. $x = -2.3$, 1.0 or 1.8 **7.** $x = -1.5$

Chapter 20

Exercise 76 (page 167)

1. 22°2′ **2.** 27°54′ **3.** 51°11′ **4.** 100°6′52″

5. 15°44′17″ **6.** 86°49′1″ **7.** 72.55° **8.** 27.754°

9. 37°57′ **10.** 58°22′52″

Exercise 77 (page 169)

1. reflex **2.** obtuse **3.** acute **4.** right angle

5. (a) 21° (b) 62°23′ (c) 48°56′17″

6. (a) 102° (b) 165° (c) 10°18′49″

7. (a) 60° (b) 110° (c) 75° (d) 143° (e) 140°
(f) 20° (g) 129.3° (h) 79° (i) 54°

8. Transversal (a) 1 & 3, 2 & 4, 5 & 7, 6 & 8
(b) 1 & 2, 2 & 3, 3 & 4, 4 & 1, 5 & 6, 6 & 7,
7 & 8, 8 & 5, 3 & 8, 1 & 6, 4 & 7 or 2 & 5
(c) 1 & 5, 2 & 6, 4 & 8, 3 & 7 (d) 3 & 5 or 2 & 8

9. 59°20′ **10.** $a = 69°, b = 21°, c = 82°$ **11.** 51°

12. 1.326 rad **13.** 0.605 rad **14.** 40°55′

Exercise 78 (page 173)

1. (a) acute-angled scalene triangle
(b) isosceles triangle (c) right-angled triangle
(d) obtuse-angled scalene triangle
(e) equilateral triangle (f) right-angled triangle

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