

The quick way to change $5\frac{3}{4}$ into an improper fraction is $\frac{4 \times 5 + 3}{4} = \frac{23}{4}$.

(b) $1\frac{7}{9} = \frac{9 \times 1 + 7}{9} = \frac{16}{9}$.

(c) $2\frac{3}{7} = \frac{7 \times 2 + 3}{7} = \frac{17}{7}$.

Problem 3. In a school there are 180 students of which 72 are girls. Express this as a fraction in its simplest form

The fraction of girls is $\frac{72}{180}$.

Dividing both the numerator and denominator by the lowest prime number, i.e. 2, gives

$$\frac{72}{180} = \frac{36}{90}$$

Dividing both the numerator and denominator again by 2 gives

$$\frac{72}{180} = \frac{36}{90} = \frac{18}{45}$$

2 will not divide into both 18 and 45 so dividing both the numerator and denominator by the next prime number, i.e. 3, gives

$$\frac{72}{180} = \frac{36}{90} = \frac{18}{45} = \frac{6}{15}$$

Dividing both the numerator and denominator again by 3 gives

$$\frac{72}{180} = \frac{36}{90} = \frac{18}{45} = \frac{6}{15} = \frac{2}{5}$$

So $\frac{72}{180} = \frac{2}{5}$ in its simplest form.

Thus, $\frac{2}{5}$ of the students are girls.

2.2 Adding and subtracting fractions

When the denominators of two (or more) fractions to be added are the same, the fractions can be added 'on sight'.

For example, $\frac{2}{9} + \frac{5}{9} = \frac{7}{9}$ and $\frac{3}{8} + \frac{1}{8} = \frac{4}{8}$.

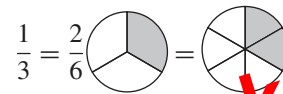
In the latter example, dividing both the 4 and the 8 by 4 gives $\frac{4}{8} = \frac{1}{2}$, which is the simplified answer. This is called **cancelling**.

Addition and subtraction of fractions is demonstrated in the following worked examples.

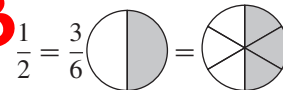
Problem 4. Simplify $\frac{1}{3} + \frac{1}{2}$

(i) Make the denominators the same for each fraction. The lowest number that both denominators divide into is called the **lowest common multiple** or **LCM** (see Chapter 1, page 5). In this example, the LCM of 3 and 2 is 6.

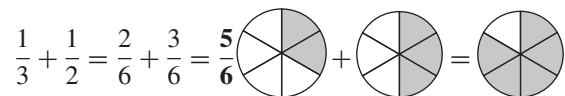
(ii) 3 divides into 6 twice. Multiplying both numerator and denominator of $\frac{1}{3}$ by 2 gives



(iii) 2 divides into 6, 3 times. Multiplying both numerator and denominator of $\frac{1}{2}$ by 3 gives



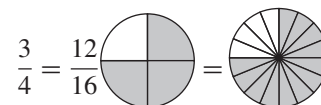
(iv) Hence,



Problem 5. Simplify $\frac{3}{4} - \frac{7}{16}$

(i) Make the denominators the same for each fraction. The lowest common multiple (LCM) of 4 and 16 is 16.

(ii) 4 divides into 16, 4 times. Multiplying both numerator and denominator of $\frac{3}{4}$ by 4 gives



(iii) $\frac{7}{16}$ already has a denominator of 16.

Chapter 10

Exercise 39 (page 69)

- | | |
|-------------------------|---------------------------|
| 1. $x^2 + 5x + 6$ | 2. $2x^2 + 9x + 4$ |
| 3. $4x^2 + 12x + 9$ | 4. $2j^2 + 2j - 12$ |
| 5. $4x^2 + 22x + 30$ | 6. $2pqr + p^2q^2 + r^2$ |
| 7. $a^2 + 2ab + b^2$ | 8. $x^2 + 12x + 36$ |
| 9. $a^2 - 2ac + c^2$ | 10. $25x^2 + 30x + 9$ |
| 11. $4x^2 - 24x + 36$ | 12. $4x^2 - 9$ |
| 13. $64x^2 + 64x + 16$ | 14. $r^2s^2 + 2rst + t^2$ |
| 15. $3ab - 6a^2$ | 16. $2x^2 - 2xy$ |
| 17. $2a^2 - 3ab - 5b^2$ | 18. $13p - 7q$ |
| 19. $7x - y - 4z$ | 20. $4a^2 - 25b^2$ |
| 21. $x^2 - 4xy + 4y^2$ | 22. $9a^2 - 6ab + b^2$ |
| 23. 0 | 24. $4 - a$ |
| 25. $4ab - 8a^2$ | 26. $3xy + 9x^2y - 15x^2$ |
| 27. $2 + 5b^2$ | 28. $11q - 2p$ |

Exercise 40 (page 71)

- | | |
|-------------------------------|--------------------------|
| 1. $2(x + 2)$ | 2. $2x(y - 4z)$ |
| 3. $p(b + 2c)$ | 4. $3x(1 - 2y)$ |
| 5. $4d(d - 3f^5)$ | 6. $4x(1 + 2x)$ |
| 7. $2x(1 + 4z)$ | 8. $x^2(1 + 3z + 5x^2)$ |
| 9. $b^2(a + b^2)$ | 10. $x(s + p + t)$ |
| 11. $3xy(xy^3 - 5y + 6)$ | 12. $2pq^2(2p^2 - 5q)$ |
| 13. $7ab(3ab - 4)$ | 14. $2xy(y + 3x + 4x^2)$ |
| 15. $2xy(x - 2y^2 + 4x^2y^3)$ | 16. $7y(4 + y + 2x)$ |
| 17. $\frac{3x}{y}$ | 18. 0 |
| | 19. $\frac{2r}{t}$ |
| 20. $(a + b)(y + 1)$ | 21. $(p + q)(x + y)$ |
| 22. $(x - y)(a + b)$ | 23. $(a - 2b)(2x + 3y)$ |

Exercise 41 (page 72)

- | | |
|-------------------------------|------------------------------|
| 1. $2x + 8x^2$ | 2. $12y^2 - 3y$ |
| 3. $4b - 15b^2$ | 4. $4 + 3a$ |
| 5. $\frac{3}{2} - 4x$ | 6. 1 |
| 7. $10y^2 - 3y + \frac{1}{4}$ | 8. $9x^2 + \frac{1}{3} - 4x$ |
| 9. $6a^2 + 5a - \frac{1}{7}$ | 10. $-15t$ |
| 11. $\frac{1}{5} - x - x^2$ | 12. $10a^2 - 3a + 2$ |

Chapter 11

Exercise 42 (page 75)

- | | | | | |
|-------|---------|------------------|--------|---------|
| 1. 1 | 2. 2 | 3. 6 | 4. -4 | 5. 2 |
| 6. 1 | 7. 2 | 8. $\frac{1}{2}$ | 9. 0 | 10. 3 |
| 11. 2 | 12. -10 | 13. 6 | 14. -2 | 15. 2.5 |
| 16. 2 | 17. 6 | 18. -3 | | |

Exercise 43 (page 76)

- | | | | | |
|---------|---------------------|--------------------|-------------|--------|
| 1. 5 | 2. -2 | 3. $-4\frac{1}{2}$ | 4. 2 | 5. 12 |
| 6. 15 | 7. -4 | 8. $5\frac{1}{3}$ | 9. 2 | 10. 13 |
| 11. -10 | 12. 2 | 13. 3 | 14. 11 | 15. -6 |
| 16. 9 | 17. $6\frac{1}{4}$ | 18. -1 | 19. 4 | 20. 10 |
| 21. -2 | 22. $-3\frac{1}{3}$ | 23. ± 3 | 24. ± 4 | |

Exercise 44 (page 79)

- | | | |
|---|----------------------|----------|
| 1. 10^{-7} | 2. 8 m/s^2 | 3. 3.472 |
| 4. (a) 1.8Ω | (b) 30Ω | |
| 5. digital camera battery £9, camcorder battery £14 | | |
| 6. 800Ω | 7. 30 m/s^2 | |

Exercise 45 (page 80)

- | | | |
|-----------------------------|--------------|---------------|
| 1. 12 cm, 240 cm^2 | 2. 0.004 | 3. 30 |
| 4. 45°C | 5. 50 | 6. £312, £240 |
| 7. 30 kg | 8. 12 m, 8 m | 9. 3.5 N |

Chapter 12

Exercise 46 (page 84)

- | | |
|-------------------------|-----------------------------|
| 1. $d = c - e - a - b$ | 2. $x = \frac{y}{7}$ |
| 3. $v = \frac{c}{p}$ | 4. $a = \frac{v - u}{t}$ |
| 5. $R = \frac{V}{I}$ | 6. $y = \frac{1}{3}(t - x)$ |
| 7. $r = \frac{c}{2\pi}$ | 8. $x = \frac{y - c}{m}$ |

5. $x = 2, y = 4, z = 5$ 6. $x = 1, y = 6, z = 7$
 7. $x = 5, y = 4, z = 2$ 8. $x = -4, y = 3, z = 2$
 9. $x = 1.5, y = 2.5, z = 4.5$
 10. $i_1 = -5, i_2 = -4, i_3 = 2$
 11. $F_1 = 2, F_2 = -3, F_3 = 4$

Chapter 14

Exercise 54 (page 104)

1. 4 or -4 2. 4 or -8 3. 2 or -6
 4. -1.5 or 1.5 5. 0 or $-\frac{4}{3}$ 6. 2 or -2
 7. 4 8. -5 9. 1
 10. -2 or -3 11. -3 or -7 12. 2 or -1
 13. 4 or -3 14. 2 or 7 15. -4
 16. 2 17. -3 18. 3 or -3
 19. -2 or $-\frac{2}{3}$ 20. -1.5 21. $\frac{1}{8}$ or $-\frac{1}{8}$
 22. 4 or -7 23. -1 or 1.5 24. $\frac{1}{2}$ or $\frac{1}{3}$
 25. $\frac{1}{2}$ or $-\frac{4}{5}$ 26. $\frac{1}{3}$ or $-\frac{1}{7}$ 27. $\frac{3}{8}$ or $-\frac{1}{4}$
 28. $\frac{2}{5}$ or -3 29. $\frac{4}{2}$ or $-\frac{3}{2}$ 30. $\frac{5}{4}$ or $-\frac{3}{4}$
 31. $x^2 - 4x + 5 = 0$ 32. $x^2 + 3x - 10 = 0$
 33. $x^2 + 5x + 4 = 0$ 34. $4x^2 - 8x - 5 = 0$
 35. $x^2 - 36 = 0$ 36. $x^2 - 1.7x - 1.68 = 0$

Exercise 55 (page 106)

1. -3.732 or -0.268 2. -3.137 or 0.637
 3. 1.468 or -1.135 4. 1.290 or 0.310
 5. 2.443 or 0.307 6. -2.851 or 0.351

Exercise 56 (page 107)

1. 0.637 or -3.137 2. 0.296 or -0.792
 3. 2.781 or 0.719 4. 0.443 or -1.693
 5. 3.608 or -1.108 6. 1.434 or 0.232
 7. 0.851 or -2.351 8. 2.086 or -0.086
 9. 1.481 or -1.081 10. 4.176 or -1.676
 11. 4 or 2.167 12. 7.141 or -3.641
 13. 4.562 or 0.438

Exercise 57 (page 109)

1. 1.191 s 2. 0.345 A or 0.905 A 3. 7.84 cm
 4. 0.619 m or 19.38 m 5. 0.0133
 6. 1.066 m 7. 86.78 cm
 8. 1.835 m or 18.165 m 9. 7 m
 10. 12 ohms, 28 ohms

Exercise 58 (page 110)

1. $x = 1, y = 3$ and $x = -3, y = 7$
 2. $x = \frac{2}{5}, y = -\frac{1}{5}$ and $-1\frac{2}{3}, y = -4\frac{1}{3}$
 3. $x = 0, y = 4$ and $x = 3, y = 1$

Chapter 15

Exercise 59 (page 112)

1. 4 2. 4 3. 3 4. -3 5. $\frac{1}{3}$
 6. 3 7. 2 8. -2 9. $1\frac{1}{2}$ 10. $\frac{1}{3}$
 11. 2 12. 10000 13. 100000 14. 9 15. $\frac{1}{32}$
 16. 0.01 17. $\frac{1}{16}$ 18. e^3

Exercise 60 (page 115)

1. log 6 2. log 15 3. log 2 4. log 3
 5. log 12 6. log 500 7. log 100 8. log 6
 9. log 10 10. log 1 = 0 11. log 2
 12. log 243 or log 3⁵ or 5 log 3
 13. log 16 or log 2⁴ or 4 log 2
 14. log 64 or log 2⁶ or 6 log 2
 15. 0.5 16. 1.5 17. $x = 2.5$ 18. $t = 8$
 19. $b = 2$ 20. $x = 2$ 21. $a = 6$ 22. $x = 5$

Exercise 61 (page 116)

1. 1.690 2. 3.170 3. 0.2696 4. 6.058 5. 2.251
 6. 3.959 7. 2.542 8. -0.3272 9. 316.2

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