The quick way to change $5\frac{3}{4}$ into an improper fraction is $\frac{4\times5+3}{4}=\frac{\mathbf{23}}{\mathbf{4}}$.

(b) $1\frac{7}{9}=\frac{9\times1+7}{9}=\frac{\mathbf{16}}{\mathbf{9}}$.

(c) $2\frac{3}{7}=\frac{7\times2+3}{7}=\frac{\mathbf{17}}{\mathbf{7}}$.

Problem 3. In a school there are 180 students of which 72 are girls. Express this as a fraction in its simplest form

The fraction of girls is $\frac{72}{180}$.
Dividing both the numerator and denominator by the lowest prime number, i.e. 2, gives

$$\frac{72}{180}=\frac{36}{90}$$

Dividing both the numerator and denominator again by 2 gives

$$\frac{72}{180}=\frac{36}{90}=\frac{18}{45}$$

2 will not divide into both 18 and 45. So dividing both the numerator and denominator by the next prime number, i.e. 3, gives

$$\frac{72}{180}=\frac{36}{90}=\frac{18}{45}=\frac{6}{15}$$

Dividing both the numerator and denominator again by 3 gives

$$\frac{72}{180}=\frac{36}{90}=\frac{18}{45}=\frac{6}{15}=\frac{2}{5}$$

So $\mathbf{\frac{72}{180}=\frac{2}{5}}$ **in its simplest form**.

Thus, $\mathbf{\frac{2}{5}}$ **of the students are girls**.

2.2 Adding and subtracting fractions

When the denominators of two (or more) fractions to be added are the same, the fractions can be added 'on sight'.

For example, $\frac{2}{9}+\frac{5}{9}=\frac{7}{9}$ and $\frac{3}{8}+\frac{1}{8}=\frac{4}{8}$.

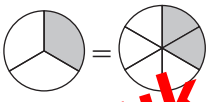
In the latter example, dividing both the 4 and the 8 by 4 gives $\frac{4}{8}=\frac{1}{2}$, which is the simplified answer. This is called **cancelling**.

Addition and subtraction of fractions is demonstrated in the following worked examples.

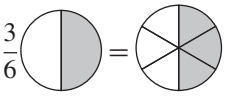
Problem 4. Simplify $\frac{1}{3}+\frac{1}{2}$

(i) Make the denominators the same for each fraction. The lowest number that both denominators divide into is called the **lowest common multiple** or **LCM** (see Chapter 1, page 5). In this example, the LCM of 3 and 2 is 6.

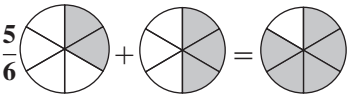
(ii) 3 divides into 6 twice. Multiplying both numerator and denominator of $\frac{1}{3}$ by 2 gives

$\frac{1}{3}=\frac{2}{6}=$ 

(iii) 2 divides into 6, 3 times. Multiplying both numerator and denominator of $\frac{1}{2}$ by 3 gives

$\frac{1}{2}=\frac{3}{6}=$ 

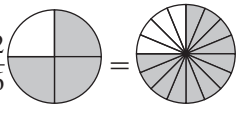
(iv) Hence,

$\frac{1}{3}+\frac{1}{2}=\frac{2}{6}+\frac{3}{6}=\mathbf{\frac{5}{6}}=$ 

Problem 5. Simplify $\frac{3}{4}-\frac{7}{16}$

(i) Make the denominators the same for each fraction. The lowest common multiple (LCM) of 4 and 16 is 16.

(ii) 4 divides into 16, 4 times. Multiplying both numerator and denominator of $\frac{3}{4}$ by 4 gives

$\frac{3}{4}=\frac{12}{16}=$ 

(iii) $\frac{7}{16}$ already has a denominator of 16.

Chapter 10

Exercise 39 (page 69)

1. x^2+5x+6 **2.** $2x^2+9x+4$
3. $4x^2+12x+9$ **4.** $2j^2+2j-12$
5. $4x^2+22x+30$ **6.** $2pqr+p^2q^2+r^2$
7. $a^2+2ab+b^2$ **8.** $x^2+12x+36$
9. $a^2-2ac+c^2$ **10.** $25x^2+30x+9$
11. $4x^2-24x+36$ **12.** $4x^2-9$
13. $64x^2+64x+16$ **14.** $r^2s^2+2rst+t^2$
15. $3ab-6a^2$ **16.** $2x^2-2xy$
17. $2a^2-3ab-5b^2$ **18.** $13p-7q$
19. $7x-y-4z$ **20.** $4a^2-25b^2$
21. $x^2-4xy+4y^2$ **22.** $9a^2-6ab+b^2$
23. 0 **24.** $4-a$
25. $4ab-8a^2$ **26.** $3xy+9x^2y-15x^2$
27. $2+5b^2$ **28.** $11q-2p$

Exercise 40 (page 71)

1. $2(x+2)$ **2.** $2x(y-4)$
3. $p(b+2c)$ **4.** $7x(1+2y)$
5. $4d(d-3f^5)$ **6.** $4x(1+2x)$
7. $2y(6-4y)$ **8.** $x(1+3x-5x^2)$
9. $ba(a+b^2)$ **10.** $-(s+p+t)$
11. $3xy(xy^3-5y+6)$ **12.** $2pq^2(2p^2-5q)$
13. $7ab(3ab-4)$ **14.** $2xy(y+3x+4x^2)$
15. $2xy\left(x-2y^2+4x^2y^3\right)$ **16.** $7y(4+y+2x)$
17. $\dfrac{3x}{y}$ **18.** 0 **19.** $\dfrac{2r}{t}$
20. $(a+b)(y+1)$ **21.** $(p+q)(x+y)$
22. $(x-y)(a+b)$ **23.** $(a-2b)(2x+3y)$

Exercise 41 (page 72)

1. $2x+8x^2$ **2.** $12y^2-3y$
3. $4b-15b^2$ **4.** $4+3a$
5. $\dfrac{3}{2}-4x$ **6.** 1
7. $10y^2-3y+\dfrac{1}{4}$ **8.** $9x^2+\dfrac{1}{3}-4x$
9. $6a^2+5a-\dfrac{1}{7}$ **10.** $-15t$
11. $\dfrac{1}{5}-x-x^2$ **12.** $10a^2-3a+2$

Chapter 11

Exercise 42 (page 75)

1. 1 **2.** 2 **3.** 6 **4.** -4 **5.** 2
6. 1 **7.** 2 **8.** $\dfrac{1}{2}$ **9.** 0 **10.** 3
11. 2 **12.** -10 **13.** 6 **14.** -2 **15.** 2.5
16. 2 **17.** 6 **18.** -3

Exercise 43 (page 76)

1. 5 **2.** -2 **3.** $-4\dfrac{1}{2}$ **4.** 2 **5.** 12
6. 15 **7.** -4 **8.** $5\dfrac{1}{3}$ **9.** 2 **10.** 13
11. -10 **12.** 2 **13.** 3 **14.** -11 **15.** -6
16. 9 **17.** $6\dfrac{1}{4}$ **18.** [illegible] **19.** 4 **20.** 10
21. ± 2 **22.** $-3\dfrac{1}{3}$ **23.** ± 3 **24.** ± 4

Exercise 44 (page 79)

1. 10^{-7} **2.** $8\,\text{m/s}^2$ **3.** 3.472
4. (a) $1.8\,\Omega$ (b) $30\,\Omega$
5. digital camera battery £9, camcorder battery £14
6. $800\,\Omega$ **7.** $30\,\text{m/s}^2$

Exercise 45 (page 80)

1. 12 cm, 240 cm^2 **2.** 0.004 **3.** 30
4. 45°C **5.** 50 **6.** £312, £240
7. 30 kg **8.** 12 m, 8 m **9.** 3.5 N

Chapter 12

Exercise 46 (page 84)

1. $d=c-e-a-b$ **2.** $x=\dfrac{y}{7}$
3. $v=\dfrac{c}{p}$ **4.** $a=\dfrac{v-u}{t}$
5. $R=\dfrac{V}{I}$ **6.** $y=\dfrac{1}{3}(t-x)$
7. $r=\dfrac{c}{2\pi}$ **8.** $x=\dfrac{y-c}{m}$

5. $x = 2, y = 4, z = 5$ **6.** $x = 1, y = 6, z = 7$
7. $x = 5, y = 4, z = 2$ **8.** $x = -4, y = 3, z = 2$
9. $x = 1.5, y = 2.5, z = 4.5$
10. $i_1 = -5, i_2 = -4, i_3 = 2$
11. $F_1 = 2, F_2 = -3\ F_3 = 4$

Chapter 14

Exercise 54 (page 104)

1. 4 or −4 **2.** 4 or −8 **3.** 2 or −6
4. −1.5 or 1.5 **5.** 0 or $-\frac{4}{3}$ **6.** 2 or −2
7. 4 **8.** −5 **9.** 1
10. −2 or −3 **11.** −3 or −7 **12.** 2 or −1
13. 4 or −3 **14.** 2 or 7 **15.** −4
16. 2 **17.** −3 **18.** 3 or −3
19. −2 or $-\frac{2}{3}$ **20.** −1.5 **21.** $\frac{1}{8}$ or $-\frac{1}{8}$
22. 4 or −7 **23.** −1 or 1.5 **24.** $\frac{1}{2}$ or $\frac{1}{3}$
25. $\frac{1}{2}$ or $-\frac{4}{5}$ **26.** $1\frac{1}{3}$ or $-\frac{1}{7}$ **27.** $\frac{3}{8}$ or $-\frac{1}{2}$
28. $\frac{2}{5}$ or −3 **29.** $\frac{4}{3}$ or $-\frac{1}{2}$ **30.** $\frac{5}{4}$ or $-\frac{3}{2}$
31. $x^2 - 4x + 3 = 0$ **32.** $x^2 + 3x - 10 = 0$
33. $x^2 + 5x + 4 = 0$ **34.** $4x^2 - 8x - 5 = 0$
35. $x^2 - 36 = 0$ **36.** $x^2 - 1.7x - 1.68 = 0$

Exercise 55 (page 106)

1. −3.732 or −0.268 **2.** −3.137 or 0.637
3. 1.468 or −1.135 **4.** 1.290 or 0.310
5. 2.443 or 0.307 **6.** −2.851 or 0.351

Exercise 56 (page 107)

1. 0.637 or −3.137 **2.** 0.296 or −0.792
3. 2.781 or 0.719 **4.** 0.443 or −1.693
5. 3.608 or −1.108 **6.** 1.434 or 0.232
7. 0.851 or −2.351 **8.** 2.086 or −0.086
9. 1.481 or −1.081 **10.** 4.176 or −1.676
11. 4 or 2.167 **12.** 7.141 or −3.641
13. 4.562 or 0.438

Exercise 57 (page 109)

1. 1.191 s **2.** 0.345 A or 0.905 A **3.** 7.84 cm
4. 0.619 m or 19.38 m **5.** 0.0133
6. 1.066 m **7.** 86.78 cm
8. 1.835 m or 18.165 m **9.** 7 m
10. 12 ohms, 28 ohms

Exercise 58 (page 110)

1. $x = 1, y = 3$ and $x = -3, y = 7$
2. $x = \frac{2}{5}, y = -\frac{1}{5}$ and $-1\frac{2}{3}, y = -4\frac{1}{3}$
3. $x = 0, y = 4$ and $x = 3, y = 1$

Chapter 15

Exercise 59 (page 112)

1. 4 **2.** 4 **3.** 3 **4.** −3 **5.** $\frac{1}{3}$
6. 3 **7.** 2 **8.** −2 **9.** $1\frac{1}{2}$ **10.** $\frac{1}{3}$
11. 2 **12.** 10 000 **13.** 100 000 **14.** 9 **15.** $\frac{1}{32}$
16. 0.01 **17.** $\frac{1}{16}$ **18.** e^3

Exercise 60 (page 115)

1. $\log 6$ **2.** $\log 15$ **3.** $\log 2$ **4.** $\log 3$
5. $\log 12$ **6.** $\log 500$ **7.** $\log 100$ **8.** $\log 6$
9. $\log 10$ **10.** $\log 1 = 0$ **11.** $\log 2$
12. $\log 243$ or $\log 3^5$ or $5\log 3$
13. $\log 16$ or $\log 2^4$ or $4\log 2$
14. $\log 64$ or $\log 2^6$ or $6\log 2$
15. 0.5 **16.** 1.5 **17.** $x = 2.5$ **18.** $t = 8$
19. $b = 2$ **20.** $x = 2$ **21.** $a = 6$ **22.** $x = 5$

Exercise 61 (page 116)

1. 1.690 **2.** 3.170 **3.** 0.2696 **4.** 6.058 **5.** 2.251
6. 3.959 **7.** 2.542 **8.** −0.3272 **9.** 316.2

This page intentionally left blank