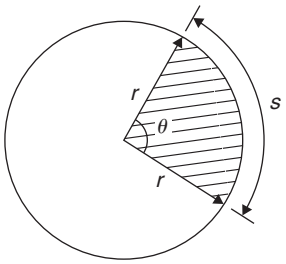
(v) **Circle** Area $= \pi r^2$ Circumference $= 2\pi r$



Radian measure: 2π radians $= 360$ degrees

For a sector of circle:

arc length, $s = \dfrac{\theta^\circ}{360}(2\pi r) = r\theta$ (θ in rad)

shaded area $= \dfrac{\theta^\circ}{360}(\pi r^2) = \dfrac{1}{2}r^2\theta$ (θ in rad)

Equation of a circle, centre at origin, radius r:

$$x^2 + y^2 = r^2$$

Equation of a circle, centre at (a, b), radius r:

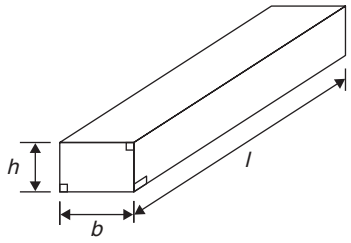
$$(x-a)^2 + (y-b)^2 = r^2$$

Volumes and surface areas of regular solids:

(i) **Rectangular prism (or cuboid)**

Volume $= l \times b \times h$

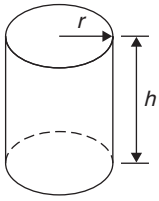
Surface area $= 2(bh + hl + lb)$



(ii) **Cylinder**

Volume $= \pi r^2 h$

Total surface area $= 2\pi rh + 2\pi r^2$

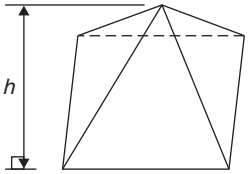


(iii) **Pyramid**

If area of base $= A$ and

perpendicular height $= h$ then:

$$\text{Volume} = \frac{1}{3} \times A \times h$$



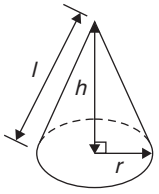
Total surface area = sum of areas of triangles forming sides + area of base

(iv) **Cone**

Volume $= \dfrac{1}{3}\pi r^2 h$

Curved Surface area $= \pi rl$

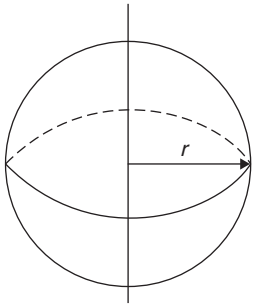
Total Surface area $= \pi rl + \pi r^2$



(v) **Sphere**

$$\text{Volume} = \frac{4}{3}\pi r^3$$

Surface area $= 4\pi r^2$



Chapter 5

Exercise 21 (page 34)

1. 0.32% **2.** 173.4% **3.** 5.7% **4.** 37.4%
5. 128.5% **6.** 0.20 **7.** 0.0125 **8.** 0.6875
9. 38.462% **10.** (a) 21.2% (b) 79.2% (c) 169%
11. (b), (d), (c), (a) **12.** $\frac{13}{20}$ **13.** $\frac{5}{16}$ **14.** $\frac{9}{16}$
15. $A = \frac{1}{2}$, $B = 50\%$, $C = 0.25$, $D = 25\%$, $E = 0.30$, $F = \frac{3}{10}$, $G = 0.60$, $H = 60\%$, $I = 0.85$, $J = \frac{17}{20}$

Exercise 22 (page 36)

1. 21.8 kg **2.** 9.72 m
3. (a) 496.4 t (b) 8.657 g (c) 20.73 s **4.** 2.25%
5. (a) 14% (b) 15.67% (c) 5.36% **6.** 37.8 g
7. 14 minutes 57 seconds **8.** 76 g **9.** £611
10. 37.49% **11.** 39.2% **12.** 17% **13.** 38.7%
14. 2.7% **15.** 5.60 m **16.** 3.5%

Exercise 23 (page 38)

1. 25% **2.** 18% **3.** £350 **4.** £175 000
5. £260 **6.** £20 000 **7.** £9176.45 **8.** £50.25
9. £39.60 **10.** £917.70 **11.** £185 000 **12.** 7.2%
13. *A* 0.6 kg, *B* 0.9 kg, *C* 0.5 kg
14. 54%, 31%, 15%, 0.3 t
15. 20 000 kg (or 20 tonnes)
16. 13.5 mm, 11.5 mm **17.** 600 kW

Chapter 6

Exercise 24 (page 41)

1. 36 : 1 **2.** 3.5 : 1 or 7 : 2 **3.** 47 : 3
4. 96 cm, 240 cm **5.** $5\frac{1}{4}$ hours or 5 hours 15 minutes
6. £3680, £1840, £920 **7.** 12 cm **8.** £2172

Exercise 25 (page 42)

1. 1 : 15 **2.** 76 ml **3.** 25% **4.** 12.6 kg
5. 14.3 kg **6.** 25 000 kg

Exercise 26 (page 43)

1. £556 **2.** £66 **3.** 264 kg **4.** 450 g **5.** 14.56 kg
6. (a) 0.00025 (b) 48 MPa **7.** (a) 440 K (b) 5.76 litre

Exercise 27 (page 45)

1. (a) 2 mA (b) 25 V **2.** 170 fr **3.** 685.8 mm
4. 83 lb 10 oz **5.** (a) 159.1 litres (b) 16.5 gallons
6. 29.4 MPa **7.** 584.2 mm **8.** $1012

Exercise 28 (page 46)

1. 3.5 weeks **2.** 20 days
3. (a) 9.18 (b) 6.12 (c) 0.3375 **4.** 50 minutes
5. (a) 300×10^3 (b) 0.375 m^2 (c) 24×10^3 Pa

Chapter 7

Exercise 29 (page 48)

1. 27 **2.** 128 **3.** 100 000 **4.** 96 **5.** 2^4
6. ± 5 **7.** ± 8 **8.** 100 **9.** 1 **10.** 64

Exercise 30 (page 50)

1. 128 **2.** 3^9 **3.** 16 **4.** $\frac{1}{9}$ **5.** 1 **6.** 8
7. 100 **8.** 1000 **9.** $\frac{1}{100}$ or 0.01 **10.** 5 **11.** 7^6
12. 3^6 **13.** 3^6 **14.** 3^4 **15.** 1 **16.** 25
17. $\frac{1}{3^5}$ or $\frac{1}{243}$ **18.** 49 **19.** $\frac{1}{2}$ or 0.5 **20.** 1

Exercise 31 (page 52)

1. $\frac{1}{3 \times 5^2}$ **2.** $\frac{1}{7^3 \times 3^7}$ **3.** $\frac{3^2}{2^5}$ **4.** $\frac{1}{2^{10} \times 5^2}$
5. 9 **6.** ± 3 **7.** $\pm\frac{1}{2}$ **8.** $\pm\frac{2}{3}$
9. $\frac{147}{148}$ **10.** $-1\frac{19}{56}$ **11.** $-3\frac{13}{45}$ **12.** $\frac{1}{9}$
13. $-\frac{17}{18}$ **14.** 64 **15.** $4\frac{1}{2}$

5. $x = 2, y = 4, z = 5$ **6.** $x = 1, y = 6, z = 7$
7. $x = 5, y = 4, z = 2$ **8.** $x = -4, y = 3, z = 2$
9. $x = 1.5, y = 2.5, z = 4.5$
10. $i_1 = -5, i_2 = -4, i_3 = 2$
11. $F_1 = 2, F_2 = -3\ F_3 = 4$

Chapter 14

Exercise 54 (page 104)

1. 4 or −4 **2.** 4 or −8 **3.** 2 or −6
4. −1.5 or 1.5 **5.** 0 or $-\frac{4}{3}$ **6.** 2 or −2
7. 4 **8.** −5 **9.** 1
10. −2 or −3 **11.** −3 or −7 **12.** 2 or −1
13. 4 or −3 **14.** 2 or 7 **15.** −4
16. 2 **17.** −3 **18.** 3 or −3
19. −2 or $-\frac{2}{3}$ **20.** −1.5 **21.** $\frac{1}{8}$ or $-\frac{1}{8}$
22. 4 or −7 **23.** −1 or 1.5 **24.** $\frac{1}{2}$ or $\frac{1}{3}$
25. $\frac{1}{2}$ or $-\frac{4}{5}$ **26.** $1\frac{1}{3}$ or $-\frac{1}{7}$ **27.** $\frac{3}{8}$ or $-\frac{1}{2}$
28. $\frac{2}{5}$ or −3 **29.** $\frac{4}{3}$ or $-\frac{1}{2}$ **30.** $\frac{5}{4}$ or $-\frac{3}{2}$
31. $x^2 - 4x + 3 = 0$ **32.** $x^2 + 3x - 10 = 0$
33. $x^2 + 5x + 4 = 0$ **34.** $4x^2 - 8x - 5 = 0$
35. $x^2 - 36 = 0$ **36.** $x^2 - 1.7x - 1.68 = 0$

Exercise 55 (page 106)

1. −3.732 or −0.268 **2.** −3.137 or 0.637
3. 1.468 or −1.135 **4.** 1.290 or 0.310
5. 2.443 or 0.307 **6.** −2.851 or 0.351

Exercise 56 (page 107)

1. 0.637 or −3.137 **2.** 0.296 or −0.792
3. 2.781 or 0.719 **4.** 0.443 or −1.693
5. 3.608 or −1.108 **6.** 1.434 or 0.232
7. 0.851 or −2.351 **8.** 2.086 or −0.086
9. 1.481 or −1.081 **10.** 4.176 or −1.676
11. 4 or 2.167 **12.** 7.141 or −3.641
13. 4.562 or 0.438

Exercise 57 (page 109)

1. 1.191 s **2.** 0.345 A or 0.905 A **3.** 7.84 cm
4. 0.619 m or 19.38 m **5.** 0.0133
6. 1.066 m **7.** 86.78 cm
8. 1.835 m or 18.165 m **9.** 7 m
10. 12 ohms, 28 ohms

Exercise 58 (page 110)

1. $x = 1, y = 3$ and $x = -3, y = 7$
2. $x = \frac{2}{5}, y = -\frac{1}{5}$ and $-1\frac{2}{3}, y = -4\frac{1}{3}$
3. $x = 0, y = 4$ and $x = 3, y = 1$

Chapter 15

Exercise 59 (page 112)

1. 4 **2.** 4 **3.** 3 **4.** −3 **5.** $\frac{1}{3}$
6. 3 **7.** 2 **8.** −2 **9.** $1\frac{1}{2}$ **10.** $\frac{1}{3}$
11. 2 **12.** 10 000 **13.** 100 000 **14.** 9 **15.** $\frac{1}{32}$
16. 0.01 **17.** $\frac{1}{16}$ **18.** e^3

Exercise 60 (page 115)

1. $\log 6$ **2.** $\log 15$ **3.** $\log 2$ **4.** $\log 3$
5. $\log 12$ **6.** $\log 500$ **7.** $\log 100$ **8.** $\log 6$
9. $\log 10$ **10.** $\log 1 = 0$ **11.** $\log 2$
12. $\log 243$ or $\log 3^5$ or $5 \log 3$
13. $\log 16$ or $\log 2^4$ or $4 \log 2$
14. $\log 64$ or $\log 2^6$ or $6 \log 2$
15. 0.5 **16.** 1.5 **17.** $x = 2.5$ **18.** $t = 8$
19. $b = 2$ **20.** $x = 2$ **21.** $a = 6$ **22.** $x = 5$

Exercise 61 (page 116)

1. 1.690 **2.** 3.170 **3.** 0.2696 **4.** 6.058 **5.** 2.251
6. 3.959 **7.** 2.542 **8.** −0.3272 **9.** 316.2

Chapter 16

Exercise 62 (page 118)

1. (a) 0.1653 (b) 0.4584 (c) 22030
2. (a) 5.0988 (b) 0.064037 (c) 40.446
3. (a) 4.55848 (b) 2.40444 (c) 8.05124
4. (a) 48.04106 (b) 4.07482 (c) −0.08286
5. 2.739 **6.** 120.7 m

Exercise 63 (page 120)

1. 2.0601 **2.** (a) 7.389 (b) 0.7408
3. $1-2x^2-\frac{8}{3}x^3-2x^4$
4. $2x^{1/2}+2x^{5/2}+x^{9/2}+\frac{1}{3}x^{13/2}+\frac{1}{12}x^{17/2}+\frac{1}{60}x^{21/2}$

Exercise 64 (page 122)

1. 3.95, 2.05 **2.** 1.65, −1.30
3. (a) 28 cm^3 (b) 116 min **4.** (a) 70°C (b) 5 minutes

Exercise 65 (page 124)

1. (a) 0.55547 (b) 0.91374 (c) 8.8941
2. (a) 2.2293 (b) −0.33154 (c) 0.13087
3. −0.4904 **4.** −0.5822 **5.** 2.197 **6.** 816.2
7. 0.8274 **8.** 11.02 **9.** 1.522 **10.** 1.485
11. 1.962 **12.** 3 **13.** 4
14. 147.9 **15.** 4.901 **16.** 3.095
17. $t=e^{b+a\ln D}=e^b e^{a\ln D}=e^b e^{\ln D^a}$ i.e. $t=e^b D^a$
18. 500 **19.** $W=PV\ln\left(\frac{U_2}{U_1}\right)$

Exercise 66 (page 127)

1. (a) 150°C (b) 100.5°C **2.** 99.21 kPa
3. (a) 29.32 volts (b) 71.31×10^{-6} s
4. (a) 1.993 m (b) 2.293 m **5.** (a) 50°C (b) 55.45 s
6. 30.37 N **7.** (a) 3.04 A (b) 1.46 s
8. 2.45 mol/cm^3 **9.** (a) 7.07 A (b) 0.966 s
10. £2424

Chapter 17

Exercise 67 (page 134)

1. (a) Horizontal axis: 1 cm = 4 V (or 1 cm = 5 V), vertical axis: 1 cm = 10 Ω
(b) Horizontal axis: 1 cm = 5 m, vertical axis: 1 cm = 0.1 V
(c) Horizontal axis: 1 cm = 10 N, vertical axis: 1 cm = 0.2 mm
2. (a) −1 (b) −8 (c) −1.5 (d) 5 **3.** 14.5
4. (a) −1.1 (b) −1.4
5. The 1010 rev/min reading should be 1070 rev/min; (a) 1000 rev/min (b) 167 V

Exercise 68 (page 140)

1. Missing values: −0.75, 0.25, 0.75, 2.25, 2.75; Gradient = $\frac{1}{2}$
2. (a) 4, −2 (b) −1, 0 (c) −3, −4 (d) 0, 4
3. (a) 2, $\frac{1}{2}$ (b) 3, $2\frac{1}{2}$ (c) $\frac{1}{24}$, $\frac{1}{2}$
4. (a) 3, −3 (b) −2, 4 (c) 3, 0 (d) 0, 7
5. (a) 2, $-\frac{1}{2}$ (b) $-\frac{2}{3}$, $-1\frac{2}{3}$ (c) $\frac{1}{18}$, 2 (d) 10, $-4\frac{2}{3}$
6. (a) $\frac{3}{5}$ (b) −4 (c) $-1\frac{5}{6}$
7. (a) and (c), (b) and (e)
8. (2, 1) **9.** (1.5, 6) **10.** (1, 2)
11. (a) 89 cm (b) 11 N (c) 2.4 (d) $l=2.4W+48$
12. $P=0.15W+3.5$ **13.** $a=-20, b=412$

Exercise 69 (page 144)

1. (a) 40°C (b) 128 Ω
2. (a) 850 rev/min (b) 77.5 V
3. (a) 0.25 (b) 12 (c) $F=0.25L+12$
(d) 89.5 N (e) 592 N (f) 212 N
4. −0.003, 8.73
5. (a) 22.5 m/s (b) 6.43 s (c) $v=0.7t+15.5$
6. $m=26.9L-0.63$
7. (a) $1.31t$ (b) 22.89% (c) $F=-0.09W+2.21$
8. (a) 96×10^9 Pa (b) 0.00022 (c) 28.8×10^6 Pa

2. $a = 40°, b = 82°, c = 66°,$
$d = 75°, e = 30°, f = 75°$

3. DF, DE **4.** 52° **5.** 122.5°

6. $\phi = 51°, x = 161°$

7. 40°, 70°, 70°, 125°, isosceles

8. $a = 18°50', b = 71°10', c = 68°, d = 90°,$
$e = 22°, f = 49°, g = 41°$

9. $a = 103°, b = 55°, c = 77°, d = 125°,$
$e = 55°, f = 22°, g = 103°, h = 77°,$
$i = 103°, j = 77°, k = 81°$

10. 17° **11.** $A = 37°, B = 60°, E = 83°$

Exercise 79 (page 176)

1. (a) congruent *BAC*, *DAC* (SAS)
(b) congruent *FGE*, *JHI* (SSS)
(c) not necessarily congruent
(d) congruent *QRT*, *SRT* (RHS)
(e) congruent *UVW*, *XZY* (ASA)

2. proof

Exercise 80 (page 178)

1. $x = 16.54\,\text{mm}$, $y = 4.18\,\text{mm}$ **2.** 9 cm, 7.79 cm

3. (a) 2.28 cm (b) 4 cm **4.** 3 m

Exercise 81 (page 180)

1–5. Constructions – see similar constructions in worked problems 30 to 33 on pages 179–180.

Chapter 21

Exercise 82 (page 182)

1. 9 cm **2.** 24 m **3.** 9.54 mm
4. 20.81 cm **5.** 7.21 m **6.** 11.18 cm
7. 24.11 mm **8.** $8^2 + 15^2 = 17^2$
9. (a) 27.20 cm each (b) 45° **10.** 20.81 km
11. 3.35 m, 10 cm **12.** 132.7 nautical miles
13. 2.94 mm **14.** 24 mm

Exercise 83 (page 185)

1. $\sin Z = \frac{9}{41}, \cos Z = \frac{40}{41}, \tan X = \frac{40}{9}, \cos X = \frac{9}{41}$

2. $\sin A = \frac{3}{5}, \cos A = \frac{4}{5}, \tan A = \frac{3}{4}, \sin B = \frac{4}{5},$
$\cos B = \frac{3}{5}, \tan B = \frac{4}{3}$

3. $\sin A = \frac{8}{17}, \tan A = \frac{8}{15}$

4. $\sin X = \frac{15}{113}, \cos X = \frac{112}{113}$

5. (a) $\frac{15}{17}$ (b) $\frac{15}{17}$ (c) $\frac{8}{15}$

6. (a) $\sin\theta = \frac{7}{25}$ (b) $\cos\theta = \frac{24}{25}$

7. (a) 9.434 (b) −0.625

Exercise 84 (page 187)

1. 2.7550 **2.** 4.846 **3.** 36.52
4. (a) 0.8660 (b) −0.1010 (c) 0.5[illegible]
5. 42.33° **6.** 15.25° **7.** [illegible].78° **8.** 7°56′
9. 31°22′ **10.** [illegible]54′ **11.** 29.05° **12.** 20°21′
13. [illegible]986 **14.** 1.803

Exercise 85 (page 189)

1. (a) 12.22 (b) 5.619 (c) 14.87 (d) 8.349
(e) 5.595 (f) 5.275

2. (a) $AC = 5.831\,\text{cm}, \angle A = 59.04°, \angle C = 30.96°$
(b) $DE = 6.928\,\text{cm}, \angle D = 30°, \angle F = 60°$
(c) $\angle J = 62°, HJ = 5.634\,\text{cm}, GH = 10.59\,\text{cm}$
(d) $\angle L = 63°, LM = 6.810\,\text{cm}, KM = 13.37\,\text{cm}$
(e) $\angle N = 26°, ON = 9.125\,\text{cm}, NP = 8.201\,\text{cm}$
(f) $\angle S = 49°, RS = 4.346\,\text{cm}, QS = 6.625\,\text{cm}$

3. 6.54 m **4.** 9.40 mm

Exercise 86 (page 192)

1. 36.15 m **2.** 48 m **3.** 249.5 m **4.** 110.1 m
5. 53.0 m **6.** 9.50 m **7.** 107.8 m
8. 9.43 m, 10.56 m **9.** 60 m

Chapter 22

Exercise 87 (page 198)

1. (a) 42.78° and 137.22° (b) 188.53° and 351.47°
2. (a) 29.08° and 330.92° (b) 123.86° and 236.14°
3. (a) 44.21° and 224.21° (b) 113.12° and 293.12°

(b) 146 cm^2 (vi) (a) 86.5 cm^3 (b) 142 cm^2
(vii) (a) 805 cm^3 (b) 539 cm^2

7. (a) 17.9 cm (b) 38.0 cm **8.** 125 cm^3
9. 10.3 m^3, 25.5 m^2 **10.** 6560 litres
11. 657.1 cm^3, 1027 cm^2 **12.** 220.7 cm^3
13. (a) 1458 litres (b) 9.77 m^2 (c) £140.45

Exercise 108 (page 255)

1. 147 cm^3, 164 cm^2 **2.** 403 cm^3, 337 cm^2
3. 10480 m^3, 1852 m^2 **4.** 1707 cm^2
5. 10.69 cm **6.** 55910 cm^3, 6051 cm^2
7. 5.14 m

Exercise 109 (page 256)

1. 8 : 125 **2.** 137.2 g

Chapter 28

Exercise 110 (page 259)

1. 4.5 square units **2.** 54.7 square units **3.** 63.33 m
4. 4.70 ha **5.** 143 m^2

Exercise 111 (page 260)

1. 42.79 m **2.** 147 m^3 **3.** 20.52 m^2

Exercise 112 (page 263)

1. (a) 2 A (b) 50 V (c) 2.5 A **2.** (a) 2.5 V (b) 3 A
3. 0.093 As, 3.1 A **4.** (a) 31.83 V (b) 0
5. 49.13 cm^2, 368.5 kPa

Chapter 29

Exercise 113 (page 266)

1. A scalar quantity has magnitude only; a vector quantity has both magnitude and direction.
2. scalar **3.** scalar **4.** vector **5.** scalar
6. scalar **7.** vector **8.** scalar **9.** vector

Exercise 114 (page 273)

1. 17.35 N at 18.00° to the 12 N force
2. 13 m/s at 22.62° to the 12 m/s velocity
3. 16.40 N at 37.57° to the 13 N force
4. 28.43 N at 129.30° to the 18 N force
5. 32.31 m at 21.80° to the 30 m displacement
6. 14.72 N at −14.72° to the 5 N force
7. 29.15 m/s at 29.04° to the horizontal
8. 9.28 N at 16.70° **9.** 6.89 m/s at 159.56°
10. 15.62 N at 26.33° to the 10 N force
11. 21.07 knots, E 9.22°S

Exercise 115 (page 276)

1. (a) 54.0 N at 78.16° (b) 45.64 N at 4.66°
2. (a) 31.71 m/s at 121.81° (b) 19.55 m/s at 8.63°

Exercise 116 (page 277)

1. 83.5 km/h at 71.6° to the vertical
2. 4 minutes 55 seconds, 60°
3. 22.79 km/h, E 9.78° N

Exercise 117 (page 277)

1. $i + j - 4k$ **2.** $4i + j - 6k$
3. $-i + 7j - k$ **4.** $5i - 10k$
5. $-3i + 2j - 8k$ **6.** $-5i + 10k$
7. $i + 1.5j - 4k$ **8.** $20.5j - 10k$
9. $3.6i + 4.4j - 6.9k$ **10.** $2i + 40j - 43k$

Chapter 30

Exercise 118 (page 279)

1. $4.5\sin(A + 63.5°)$
2. (a) $20.9\sin(\omega t + 0.62)$ volts
(b) $12.5\sin(\omega t - 1.33)$ volts
3. $13\sin(\omega t + 0.395)$

Exercise 119 (page 281)

1. $4.5\sin(A + 63.5°)$
2. (a) $20.9\sin(\omega t + 0.62)$ volts
(b) $12.5\sin(\omega t - 1.33)$ volts
3. $13\sin(\omega t + 0.395)$

Exercise 120 (page 283)

1. $4.5\sin(A + 63.5°)$
2. (a) $20.9\sin(\omega t + 0.62)$ volts
(b) $12.5\sin(\omega t - 1.33)$ volts
3. $13\sin(\omega t + 0.395)$ **4.** $11.11\sin(\omega t + 0.324)$
5. $8.73\sin(\omega t - 0.173)$