Now try the following Practice Exercise

Practice Exercise 110 Areas of irregular figures (answers on page 352)

1. Plot a graph of $y = 3x - x^2$ by completing a table of values of y from $x = 0$ to $x = 3$. Determine the area enclosed by the curve, the x-axis and ordinates $x = 0$ and $x = 3$ by (a) the trapezoidal rule (b) the mid-ordinate rule (c) Simpson's rule.

2. Plot the graph of $y = 2x^2 + 3$ between $x = 0$ and $x = 4$. Estimate the area enclosed by the curve, the ordinates $x = 0$ and $x = 4$ and the x-axis by an approximate method.

3. The velocity of a car at one second intervals is given in the following table.

Time t (s)	0	1	2	3	4	5	6
Velocity v (m/s)	0	2.0	4.5	8.0	14.0	21.0	29.0

Determine the distance travelled in 6 seconds (i.e. the area under the v/t graph) using Simpson's rule.

4. The shape of a piece of land is shown in Figure 28.4. To estimate the area of the land, a surveyor takes measurements at intervals of 50 m, perpendicular to the straight portion with the results shown (the dimensions being in metres). Estimate the area of the land in hectares ($1\,\text{ha} = 10^4\,\text{m}^2$).

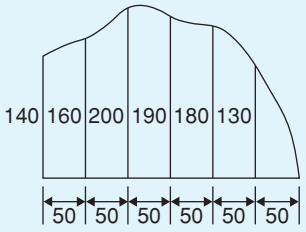

Figure 28.4

5. The deck of a ship is 35 m long. At equal intervals of 5 m the width is given by the following table.

Width (m)	0	2.8	5.2	6.5	5.8	4.1	3.0	2.3

Estimate the area of the deck.

28.2 Volumes of irregular solids

If the cross-sectional areas $A_1, A_2, A_3, \ldots$ of an irregular solid bounded by two parallel planes are known at equal intervals of width d (as shown in Figure 28.5), by Simpson's rule

$$\textbf{Volume, } V = \frac{d}{3}[(A_1 + A_7) + 4(A_2 + A_4 + A_6) + 2(A_3 + A_5)]$$

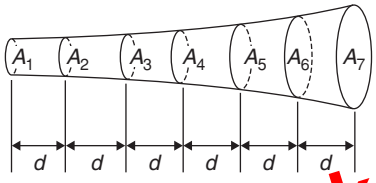

Figure 28.5

Problem 3. A tree trunk is 12 m in length and has a varying cross-section. The cross-sectional areas at intervals of 2 m measured from one end are 0.52, 0.55, 0.59, 0.63, 0.72, 0.84 and 0.97 m^2. Estimate the volume of the tree trunk

A sketch of the tree trunk is similar to that shown in Figure 28.5 above, where $d = 2\,\text{m}$, $A_1 = 0.52\,\text{m}^2$, $A_2 = 0.55\,\text{m}^2$, and so on.
Using Simpson's rule for volumes gives

$$\textbf{Volume} = \frac{2}{3}[(0.52 + 0.97) + 4(0.55 + 0.63 + 0.84) + 2(0.59 + 0.72)]$$

$$= \frac{2}{3}[1.49 + 8.08 + 2.62] = \mathbf{8.13\,m^3}$$

Problem 4. The areas of seven horizontal cross-sections of a water reservoir at intervals of 10 m are 210, 250, 320, 350, 290, 230 and 170 m^2. Calculate the capacity of the reservoir in litres

Using Simpson's rule for volumes gives

$$\text{Volume} = \frac{10}{3}[(210 + 170) + 4(250 + 350 + 230) + 2(320 + 290)]$$

$$= \frac{10}{3}[380 + 3320 + 1220] = \mathbf{16\,400\,m^3}$$

Problem 5. Determine the average values over half a cycle of the periodic waveforms shown in Figure 28.8

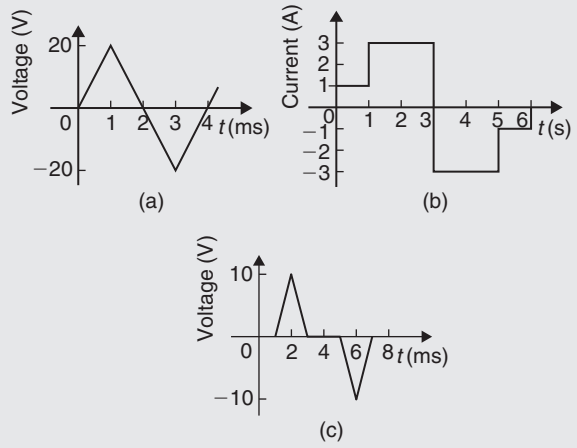

Figure 28.8

(a) Area under triangular waveform (a) for a half cycle is given by

$$\text{Area} = \frac{1}{2}(\text{base})(\text{perpendicular height})$$

$$= \frac{1}{2}(2\times10^{-3})(20) = 20\times10^{-3}\,\text{Vs}$$

Average value of waveform

$$= \frac{\text{area under curve}}{\text{length of base}}$$

$$= \frac{20\times10^{-3}\text{Vs}}{2\times10^{-3}\text{s}} = \mathbf{10\,V}$$

(b) Area under waveform (b) for a half cycle $= (1\times1)+(3\times2) = 7\,\text{As}$

$$\textbf{Average value of waveform} = \frac{\text{area under curve}}{\text{length of base}}$$

$$= \frac{7\,\text{As}}{3\,\text{s}} = \mathbf{2.33\,A}$$

(c) A half cycle of the voltage waveform (c) is completed in 4 ms.

$$\text{Area under curve} = \frac{1}{2}\{(3-1)10^{-3}\}(10)$$

$$= 10\times10^{-3}\,\text{Vs}$$

$$\textbf{Average value of waveform} = \frac{\text{area under curve}}{\text{length of base}}$$

$$= \frac{10\times10^{-3}\text{Vs}}{4\times10^{-3}\text{s}} = \mathbf{2.5\,V}$$

Problem 6. Determine the mean value of current over one complete cycle of the periodic waveforms shown in Figure 28.9

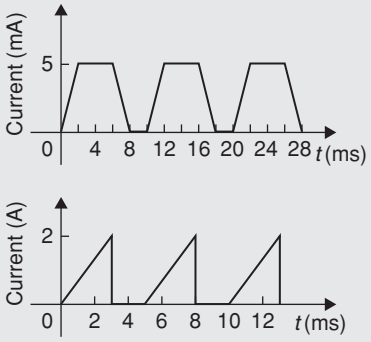

Figure 28.9

(a) One cycle of the trapezoidal waveform (a) is completed in 10 ms (i.e. the periodic time is 10 ms).

Area under curve = area of trapezium

$$= \frac{1}{2}(\text{sum of parallel sides})(\text{perpendicular distance between parallel sides})$$

$$= \frac{1}{2}\{(4+8)\times10^{-3}\}(5\times10^{-3})$$

$$= 30\times10^{-6}\,\text{As}$$

$$\textbf{Mean value over one cycle} = \frac{\text{area under curve}}{\text{length of base}}$$

$$= \frac{30\times10^{-6}\,\text{As}}{10\times10^{-3}\,\text{s}}$$

$$= \mathbf{3\,mA}$$

(b) One cycle of the saw-tooth waveform (b) is completed in 5 ms.

$$\text{Area under curve} = \frac{1}{2}(3\times10^{-3})(2)$$

$$= 3\times10^{-3}\,\text{As}$$

$$\textbf{Mean value over one cycle} = \frac{\text{area under curve}}{\text{length of base}}$$

$$= \frac{3\times10^{-3}\,\text{As}}{5\times10^{-3}\,\text{s}}$$

$$= \mathbf{0.6\,A}$$

Revision Test 11 : Volumes, irregular areas and volumes, and mean values

This assignment covers the material contained in Chapters 27 and 28. *The marks available are shown in brackets at the end of each question.*

1. A rectangular block of alloy has dimensions of 60 mm by 30 mm by 12 mm. Calculate the volume of the alloy in cubic centimetres. (3)

2. Determine how many cubic metres of concrete are required for a 120 m long path, 400 mm wide and 10 cm deep. (3)

3. Find the volume of a cylinder of radius 5.6 cm and height 15.5 cm. Give the answer correct to the nearest cubic centimetre. (3)

4. A garden roller is 0.35 m wide and has a diameter of 0.20 m. What area will it roll in making 40 revolutions? (4)

5. Find the volume of a cone of height 12.5 cm and base diameter 6.0 cm, correct to 1 decimal place. (3)

6. Find (a) the volume and (b) the total surface area of the right-angled triangular prism shown in Figure RT11.1. (9)

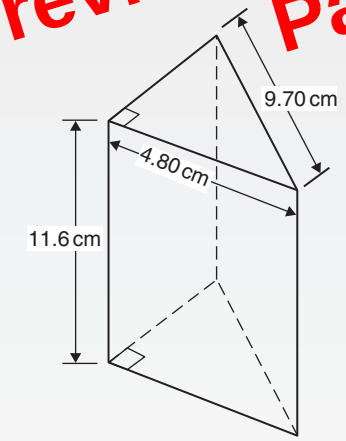

Figure RT11.1

7. A pyramid having a square base has a volume of 86.4 cm^3. If the perpendicular height is 20 cm, determine the length of each side of the base. (4)

8. A copper pipe is 80 m long. It has a bore of 80 mm and an outside diameter of 100 mm. Calculate, in cubic metres, the volume of copper in the pipe. (4)

9. Find (a) the volume and (b) the surface area of a sphere of diameter 25 mm. (4)

10. A piece of alloy with dimensions 25 mm by 60 mm by 1.60 m is melted down and recast into a cylinder whose diameter is 150 mm. Assuming no wastage, calculate the height of the cylinder in centimetres, correct to 1 decimal place. (4)

11. Determine the volume (in cubic metres) and the total surface area (in square metres) of a solid metal cone of base radius 0.5 m and perpendicular height 1.20 m. Give answers correct to 2 decimal places. (6)

12. A rectangular storage container has dimensions 3.2 m by 0.4 m by 60 cm. Determine its volume in (a) m^3 (b) cm^3. (4)

13. Calculate (a) the volume and (b) the total surface area of a 10 cm by 15 cm rectangular pyramid of height 20 cm. (8)

14. A water container is of the form of a central cylindrical part 3.0 m long and diameter 1.0 m, with a hemispherical section surmounted at each end as shown in Figure RT11.2. Determine the maximum capacity of the container, correct to the nearest litre. (1 litre = 1000 cm^3.)

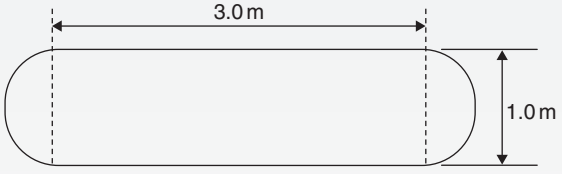

Figure RT11.2

(5)

15. Find the total surface area of a bucket consisting of an inverted frustum of a cone of slant height 35.0 cm and end diameters 60.0 cm and 40.0 cm. (4)

16. A boat has a mass of 20 000 kg. A model of the boat is made to a scale of 1 to 80. If the model is made of the same material as the boat, determine the mass of the model (in grams). (3)

9. $T = \dfrac{I}{PR}$ **10.** $L = \dfrac{X_L}{2\pi f}$

11. $R = \dfrac{E}{I}$ **12.** $x = a(y-3)$

13. $C = \dfrac{5}{9}(F-32)$ **14.** $f = \dfrac{1}{2\pi CX_C}$

Exercise 47 (page 87)

1. $r = \dfrac{S-a}{S}$ or $1 - \dfrac{a}{S}$

2. $x = \dfrac{d}{\lambda}(y+\lambda)$ or $d + \dfrac{yd}{\lambda}$

3. $f = \dfrac{3F-AL}{3}$ or $f = F - \dfrac{AL}{3}$

4. $D = \dfrac{AB^2}{5Ey}$ **5.** $t = \dfrac{R-R_0}{R_0\alpha}$ **6.** $R_2 = \dfrac{RR_1}{R_1-R}$

7. $R = \dfrac{E-e-Ir}{I}$ or $R = \dfrac{E-e}{I} - r$

8. $b = \sqrt{\left(\dfrac{y}{4ac^2}\right)}$ **9.** $x = \dfrac{ay}{\sqrt{(y^2-b^2)}}$

10. $L = \dfrac{t^2 g}{4\pi^2}$ **11.** $u = \sqrt{v^2 - 2as}$

12. $R = \sqrt{\left(\dfrac{360A}{\pi\theta}\right)}$ **13.** $y = N^2 y - x$

14. $L = \dfrac{\sqrt{Z^2-R^2}}{2\pi f}, 0.080$

Exercise 48 (page 89)

1. $a = \sqrt{\left(\dfrac{xy}{m-n}\right)}$ **2.** $R = \sqrt[4]{\left(\dfrac{M}{\pi} + r^4\right)}$

3. $r = \dfrac{3(x+y)}{(1-x-y)}$ **4.** $L = \dfrac{mrCR}{\mu - m}$

5. $b = \dfrac{c}{\sqrt{1-a^2}}$ **6.** $r = \sqrt{\left(\dfrac{x-y}{x+y}\right)}$

7. $b = \dfrac{a(p^2-q^2)}{2(p^2+q^2)}$ **8.** $v = \dfrac{uf}{u-f}, 30$

9. $t_2 = t_1 + \dfrac{Q}{mc}, 55$ **10.** $v = \sqrt{\left(\dfrac{2dgh}{0.03L}\right)}, 0.965$

11. $L = \dfrac{8S^2}{3d} + d, 2.725$

12. $C = \dfrac{1}{\omega\left\{\omega L - \sqrt{Z^2-R^2}\right\}}, 63.1 \times 10^{-6}$

13. 64 mm **14.** $\lambda = \sqrt[5]{\left(\dfrac{a\mu}{\rho CZ^4 n}\right)^2}$

Chapter 13

Exercise 49 (page 92)

1. $x = 4, y = 2$ **2.** $x = 3, y = 4$

3. $x = 2, y = 1.5$ **4.** $x = 4, y = 1$

5. $p = 2, q = -1$ **6.** $x = 1, y = 2$

7. $x = 3, y = 2$ **8.** $a = 2, b = 3$

9. $a = 5, b = 2$ **10.** $x = 1, y = 1$

11. $s = 2, t = 3$ **12.** $x = 3, y = -2$

13. $m = 2.5, n = 0.5$ **14.** $a = 6, b = -1$

15. $x = 2,\ y = 5$ **16.** $c = 2, d = -3$

Exercise 50 (page 94)

1. $p = -1, q = -2$ **2.** $x = 4, y = 6$

3. $a = 2, b = 3$ **4.** $s = 4, t = -1$

5. $x = 3, y = 4$ **6.** $u = 12, v = 2$

7. $x = 10, y = 15$ **8.** $a = 0.30, b = 0.40$

Exercise 51 (page 96)

1. $x = \dfrac{1}{2}, y = \dfrac{1}{4}$ **2.** $a = \dfrac{1}{3}, b = -\dfrac{1}{2}$

3. $p = \dfrac{1}{4}, q = \dfrac{1}{5}$ **4.** $x = 10, y = 5$

5. $c = 3, d = 4$ **6.** $r = 3, s = \dfrac{1}{2}$

7. $x = 5, y = 1\dfrac{3}{4}$ **8.** 1

Exercise 52 (page 99)

1. $a = 0.2, b = 4$ **2.** $I_1 = 6.47, I_2 = 4.62$

3. $u = 12, a = 4, v = 26$ **4.** £15 500, £12 800

5. $m = -0.5, c = 3$

6. $\alpha = 0.00426, R_0 = 22.56\,\Omega$ **7.** $a = 12, b = 0.40$

8. $a = 4, b = 10$ **9.** $F_1 = 1.5, F_2 = -4.5$

Exercise 53 (page 100)

1. $x = 2, y = 1, z = 3$ **2.** $x = 2, y = -2, z = 2$

3. $x = 5, y = -1, z = -2$ **4.** $x = 4, y = 0, z = 3$

Exercise 121 (page 284)

1. $11.11\sin(\omega t + 0.324)$ **2.** $8.73\sin(\omega t - 0.173)$
3. $i = 21.79\sin(\omega t - 0.639)$
4. $v = 5.695\sin(\omega t + 0.670)$
5. $x = 14.38\sin(\omega t + 1.444)$
6. (a) $305.3\sin(314.2t - 0.233)$ V (b) 50 Hz
7. (a) $10.21\sin(628.3t + 0.818)$ V (b) 100 Hz (c) 10 ms
8. (a) $79.83\sin(300\pi t + 0.352)$ V (b) 150 Hz (c) 6.667 ms

Chapter 31

Exercise 122 (page 288)

1. (a) continuous (b) continuous (c) discrete (d) continuous
2. (a) discrete (b) continuous (c) discrete (d) discrete

Exercise 123 (page 292)

1. If one symbol is used to represent 10 vehicles, working correct to the nearest 5 vehicles, gives 3.5, 4.5, 6, 7, 5 and 4 symbols respectively.
2. If one symbol represents 200 components, working correct to the nearest 100 components gives: Mon 8, Tues 11, Wed 6, Thurs 12 and Fri 6.5.
3. 6 equally spaced horizontal rectangles, whose lengths are proportional to 35, 44, 62, 68, 49 and 41, respectively.
4. 5 equally spaced horizontal rectangles, whose lengths are proportional to 1580, 2190, 1840, 2385 and 1280 units, respectively.
5. 6 equally spaced vertical rectangles, whose heights are proportional to 35, 44, 62, 68, 49 and 41 units, respectively.
6. 5 equally spaced vertical rectangles, whose heights are proportional to 1580, 2190, 1840, 2385 and 1280 units, respectively.
7. Three rectangles of equal height, subdivided in the percentages shown in the columns of the question. P increases by 20% at the expense of Q and R.
8. Four rectangles of equal height, subdivided as follows: week 1: 18%, 7%, 35%, 12%, 28%; week 2: 20%, 8%, 32%, 13%, 27%; week 3: 22%, 10%, 29%, 14%, 25%; week 4: 20%, 9%, 27%, 19%, 25%. Little change in centres A and B, a reduction of about 8% in C, an increase of about 7% in D and a reduction of about 3% in E.
9. A circle of any radius, subdivided into sectors having angles of 7.5°, 22.5°, 52.5°, 167.5° and 110°, respectively.
10. A circle of any radius, subdivided into sectors having angles of 107°, 156°, 29° and 68°, respectively.
11. (a) £495 (b) 88 **12.** (a) £16 450 (b) 138

Exercise 124 (page 297)

1. There is no unique solution, but one solution is:
39.3–39.4 1; 39.5–39.6 5; 39.7–39.8 9; 39.9–40.0 17; 40.1–40.2 15; 40.3–40.4 7; 40.5–40.6 4; 40.7–40.8 2.
2. Rectangles, touching one another, having midpoints of 39.35, 39.55, 39.75, 39.95, ... and heights of 1, 5, 9, 17, ...
3. There is no unique solution, but one solution is:
20.5–20.9 3; 21.0–21.4 10; 21.5–21.9 11; 22.0–22.4 13; 22.5–22.9 9; 23.0–23.4 2.
4. There is no unique solution, but one solution is:
1–10 3; 11–19 7; 20–22 12; 23–25 11; 26–28 10; 29–38 5; 39–48 2.
5. 20.95 3; 21.45 13; 21.95 24; 22.45 37; 22.95 46; 23.45 48
6. Rectangles, touching one another, having midpoints of 5.5, 15, 21, 24, 27, 33.5 and 43.5. The heights of the rectangles (frequency per unit class range) are 0.3, 0.78, 4, 4.67, 2.33, 0.5 and 0.2.
7. (10.95 2), (11.45 9), (11.95 19), (12.45 31), (12.95 42), (13.45, 50)
8. A graph of cumulative frequency against upper class boundary having co-ordinates given in the answer to problem 7.
9. (a) There is no unique solution, but one solution is:
2.05–2.09 3; 2.10–2.14 10; 2.15–2.19 11; 2.20–2.24 13; 2.25–2.29 9; 2.30–2.34 2.
(b) Rectangles, touching one another, having midpoints of 2.07, 2.12, ... and heights of 3, 10, ...
(c) Using the frequency distribution given in the solution to part (a) gives 2.095 3; 2.145 13; 2.195 24; 2.245 37; 2.295 46; 2.345 48.
(d) A graph of cumulative frequency against upper class boundary having the co-ordinates given in part (c).

Chapter 32

Exercise 125 (page 300)

1. Mean 7.33, median 8, mode 8
2. Mean 27.25, median 27, mode 26