

Now try the following Practice Exercise

Practice Exercise 110 Areas of irregular figures (answers on page 352)

1. Plot a graph of $y = 3x - x^2$ by completing a table of values of y from $x = 0$ to $x = 3$. Determine the area enclosed by the curve, the x -axis and ordinates $x = 0$ and $x = 3$ by (a) the trapezoidal rule (b) the mid-ordinate rule (c) Simpson's rule.
2. Plot the graph of $y = 2x^2 + 3$ between $x = 0$ and $x = 4$. Estimate the area enclosed by the curve, the ordinates $x = 0$ and $x = 4$ and the x -axis by an approximate method.
3. The velocity of a car at one second intervals is given in the following table.

Time t (s)	0	1	2	3	4	5	6
Velocity v (m/s)	0	2.0	4.5	8.0	14.0	21.0	29.0

Determine the distance travelled in 6 seconds (i.e. the area under the v/t graph) using Simpson's rule.

4. The shape of a piece of land is shown in Figure 28.4. To estimate the area of the land, a surveyor takes measurements at intervals of 50 m, perpendicular to the straight portion with the results shown (the dimensions being in metres). Estimate the area of the land in hectares (1 ha = 10^4 m²).

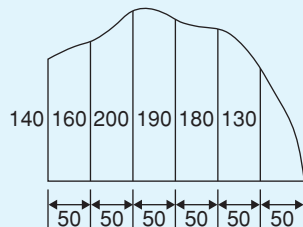


Figure 28.4

5. The deck of a ship is 35 m long. At equal intervals of 5 m the width is given by the following table.

Width (m)	0	2.8	5.2	6.5	5.8	4.1	3.0	2.3
-----------	---	-----	-----	-----	-----	-----	-----	-----

Estimate the area of the deck.

28.2 Volumes of irregular solids

If the cross-sectional areas $A_1, A_2, A_3, \dots$ of an irregular solid bounded by two parallel planes are known at equal intervals of width d (as shown in Figure 28.5), by Simpson's rule

$$\text{Volume, } V = \frac{d}{3} [(A_1 + A_7) + 4(A_2 + A_4 + A_6) + 2(A_3 + A_5)]$$

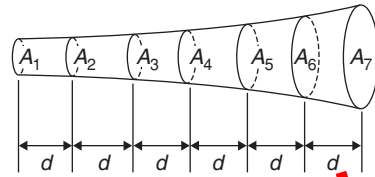


Figure 28.5

Problem 3. A tree trunk is 12 m in length and has a varying cross-section. The cross-sectional areas at intervals of 2 m measured from one end are 0.52, 0.55, 0.59, 0.63, 0.72, 0.84 and 0.97 m². Estimate the volume of the tree trunk

A sketch of the tree trunk is similar to that shown in Figure 28.5 above, where $d = 2$ m, $A_1 = 0.52$ m², $A_2 = 0.55$ m², and so on.

Using Simpson's rule for volumes gives

$$\begin{aligned} \text{Volume} &= \frac{2}{3} [(0.52 + 0.97) + 4(0.55 + 0.63 + 0.84) \\ &\quad + 2(0.59 + 0.72)] \\ &= \frac{2}{3} [1.49 + 8.08 + 2.62] = \mathbf{8.13 \text{ m}^3} \end{aligned}$$

Problem 4. The areas of seven horizontal cross-sections of a water reservoir at intervals of 10 m are 210, 250, 320, 350, 290, 230 and 170 m². Calculate the capacity of the reservoir in litres

Using Simpson's rule for volumes gives

$$\begin{aligned} \text{Volume} &= \frac{10}{3} [(210 + 170) + 4(250 + 350 + 230) \\ &\quad + 2(320 + 290)] \\ &= \frac{10}{3} [380 + 3320 + 1220] = \mathbf{16\,400 \text{ m}^3} \end{aligned}$$

Problem 5. Determine the average values over half a cycle of the periodic waveforms shown in Figure 28.8

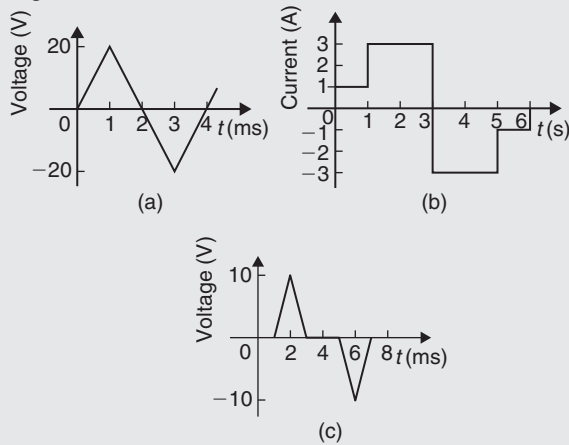


Figure 28.8

- (a) Area under triangular waveform (a) for a half cycle is given by

$$\begin{aligned} \text{Area} &= \frac{1}{2}(\text{base})(\text{perpendicular height}) \\ &= \frac{1}{2}(2 \times 10^{-3})(20) = 20 \times 10^{-3} \text{ Vs} \end{aligned}$$

Average value of waveform

$$\begin{aligned} &= \frac{\text{area under curve}}{\text{length of base}} \\ &= \frac{20 \times 10^{-3} \text{ Vs}}{2 \times 10^{-3} \text{ s}} = 10 \text{ V} \end{aligned}$$

- (b) Area under waveform (b) for a half cycle = $(1 \times 1) + (3 \times 2) = 7 \text{ As}$

$$\begin{aligned} \text{Average value of waveform} &= \frac{\text{area under curve}}{\text{length of base}} \\ &= \frac{7 \text{ As}}{3 \text{ s}} = 2.33 \text{ A} \end{aligned}$$

- (c) A half cycle of the voltage waveform (c) is completed in 4 ms.

$$\begin{aligned} \text{Area under curve} &= \frac{1}{2}\{(3 - 1)10^{-3}\}(10) \\ &= 10 \times 10^{-3} \text{ Vs} \end{aligned}$$

$$\begin{aligned} \text{Average value of waveform} &= \frac{\text{area under curve}}{\text{length of base}} \\ &= \frac{10 \times 10^{-3} \text{ Vs}}{4 \times 10^{-3} \text{ s}} = 2.5 \text{ V} \end{aligned}$$

Problem 6. Determine the mean value of current over one complete cycle of the periodic waveforms shown in Figure 28.9

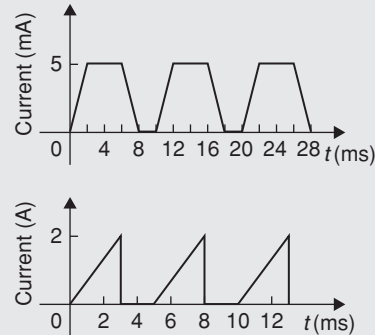


Figure 28.9

- (a) One cycle of the trapezoidal waveform (a) is completed in 10 ms (i.e. the period is 10 ms).

Area under curve = area of trapezium

$$\begin{aligned} &= \frac{1}{2}(\text{sum of parallel sides})(\text{perpendicular distance between parallel sides}) \\ &= \frac{1}{2}\{(4 + 8) \times 10^{-3}\}(5 \times 10^{-3}) \\ &= 30 \times 10^{-6} \text{ As} \end{aligned}$$

$$\begin{aligned} \text{Mean value over one cycle} &= \frac{\text{area under curve}}{\text{length of base}} \\ &= \frac{30 \times 10^{-6} \text{ As}}{10 \times 10^{-3} \text{ s}} \\ &= 3 \text{ mA} \end{aligned}$$

- (b) One cycle of the saw-tooth waveform (b) is completed in 5 ms.

$$\begin{aligned} \text{Area under curve} &= \frac{1}{2}(3 \times 10^{-3})(2) \\ &= 3 \times 10^{-3} \text{ As} \end{aligned}$$

$$\begin{aligned} \text{Mean value over one cycle} &= \frac{\text{area under curve}}{\text{length of base}} \\ &= \frac{3 \times 10^{-3} \text{ As}}{5 \times 10^{-3} \text{ s}} \\ &= 0.6 \text{ A} \end{aligned}$$

Revision Test 11 : Volumes, irregular areas and volumes, and mean values

This assignment covers the material contained in Chapters 27 and 28. The marks available are shown in brackets at the end of each question.

1. A rectangular block of alloy has dimensions of 60 mm by 30 mm by 12 mm. Calculate the volume of the alloy in cubic centimetres. (3)
2. Determine how many cubic metres of concrete are required for a 120 m long path, 400 mm wide and 10 cm deep. (3)
3. Find the volume of a cylinder of radius 5.6 cm and height 15.5 cm. Give the answer correct to the nearest cubic centimetre. (3)
4. A garden roller is 0.35 m wide and has a diameter of 0.20 m. What area will it roll in making 40 revolutions? (4)
5. Find the volume of a cone of height 12.5 cm and base diameter 6.0 cm, correct to 1 decimal place. (3)
6. Find (a) the volume and (b) the total surface area of the right-angled triangular prism shown in Figure RT11.1. (9)
7. A pyramid having a square base has a volume of 86.4 cm^3 . If the perpendicular height is 20 cm, determine the length of each side of the base. (4)
8. A copper pipe is 80 m long. It has a bore of 80 mm and an outside diameter of 100 mm. Calculate, in cubic metres, the volume of copper in the pipe. (4)
9. Find (a) the volume and (b) the surface area of a sphere of diameter 25 mm. (4)
10. A piece of alloy with dimensions 25 mm by 60 mm by 1.60 m is melted down and recast into a cylinder whose diameter is 150 mm. Assuming no wastage, calculate the height of the cylinder in centimetres, correct to 1 decimal place. (4)
11. Determine the volume (in cubic metres) and the total surface area (in square metres) of a solid metal cone of base radius 0.5 m and perpendicular height 1.20 m. Give answers correct to 2 decimal places. (6)
12. A rectangular storage container has dimensions 3.2 m by 1.0 m by 60 cm. Determine its volume in m^3 (a) and cm^3 (b). (4)
13. Calculate (a) the volume and (b) the total surface area of a 10 cm by 15 cm rectangular pyramid of height 20 cm. (8)
14. A water container is of the form of a central cylindrical part 3.0 m long and diameter 1.0 m, with a hemispherical section surmounted at each end as shown in Figure RT11.2. Determine the maximum capacity of the container, correct to the nearest litre. (1 litre = 1000 cm^3). (5)

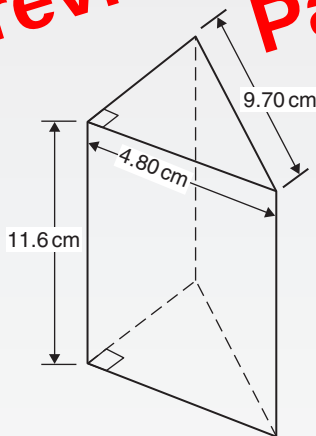


Figure RT11.1

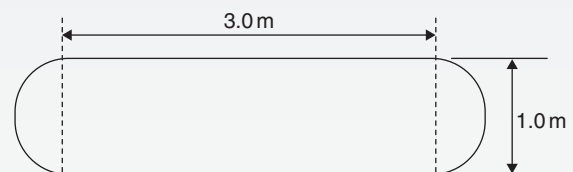


Figure RT11.2

15. Find the total surface area of a bucket consisting of an inverted frustum of a cone of slant height 35.0 cm and end diameters 60.0 cm and 40.0 cm. (4)
16. A boat has a mass of 20000 kg. A model of the boat is made to a scale of 1 to 80. If the model is made of the same material as the boat, determine the mass of the model (in grams). (3)

$$\begin{array}{ll} 9. T = \frac{I}{PR} & 10. L = \frac{X_L}{2\pi f} \\ 11. R = \frac{E}{I} & 12. x = a(y-3) \\ 13. C = \frac{5}{9}(F-32) & 14. f = \frac{1}{2\pi CX_C} \end{array}$$

Exercise 47 (page 87)

$$\begin{array}{ll} 1. r = \frac{S-a}{S} \text{ or } 1 - \frac{a}{S} & \\ 2. x = \frac{d}{\lambda}(y+\lambda) \text{ or } d + \frac{yd}{\lambda} & \\ 3. f = \frac{3F-AL}{3} \text{ or } f = F - \frac{AL}{3} & \\ 4. D = \frac{AB^2}{5Ey} & 5. t = \frac{R-R_0}{R_0\alpha} \quad 6. R_2 = \frac{RR_1}{R_1-R} \\ 7. R = \frac{E-e-Ir}{I} \text{ or } R = \frac{E-e}{I} - r & \\ 8. b = \sqrt{\left(\frac{y}{4ac^2}\right)} & 9. x = \frac{ay}{\sqrt{(y^2-b^2)}} \\ 10. L = \frac{t^2g}{4\pi^2} & 11. t = \sqrt{v^2-2as} \\ 12. R = \sqrt{\left(\frac{360A}{\pi\theta}\right)} & 13. y = N^2y-x \\ 14. L = \frac{\sqrt{Z^2-R^2}}{2\pi f}, 0.080 & \end{array}$$

Exercise 48 (page 89)

$$\begin{array}{ll} 1. a = \sqrt{\left(\frac{xy}{m-n}\right)} & 2. R = \sqrt[4]{\left(\frac{M}{\pi} + r^4\right)} \\ 3. r = \frac{3(x+y)}{(1-x-y)} & 4. L = \frac{mrCR}{\mu-m} \\ 5. b = \frac{c}{\sqrt{1-a^2}} & 6. r = \sqrt{\left(\frac{x-y}{x+y}\right)} \\ 7. b = \frac{a(p^2-q^2)}{2(p^2+q^2)} & 8. v = \frac{uf}{u-f}, 30 \\ 9. t_2 = t_1 + \frac{Q}{mc}, 55 & 10. v = \sqrt{\left(\frac{2dgh}{0.03L}\right)}, 0.965 \\ 11. L = \frac{8S^2}{3d} + d, 2.725 & \end{array}$$

$$\begin{array}{l} 12. C = \frac{1}{\omega\left\{\omega L - \sqrt{Z^2 - R^2}\right\}}, 63.1 \times 10^{-6} \\ 13. 64 \text{ mm} \quad 14. \lambda = \sqrt[5]{\left(\frac{a\mu}{\rho CZ^4n}\right)^2} \end{array}$$

Chapter 13

Exercise 49 (page 92)

$$\begin{array}{ll} 1. x=4, y=2 & 2. x=3, y=4 \\ 3. x=2, y=1.5 & 4. x=4, y=1 \\ 5. p=2, q=-1 & 6. x=1, y=2 \\ 7. x=3, y=2 & 8. a=2, b=3 \\ 9. a=5, b=2 & 10. x=1, y=1 \\ 11. s=2, t=3 & 12. x=3, y=-2 \\ 13. m=2.5, n=0.5 & 14. a=6, b=-1 \\ 15. x=2, y=5 & 16. c=2, d=-1 \end{array}$$

Exercise 50 (page 94)

$$\begin{array}{ll} 1. p=-1, q=-2 & 2. x=4, y=6 \\ 3. a=2, b=-1 & 4. s=4, t=-1 \\ 5. x=6, y=4 & 6. u=12, v=2 \\ 7. x=10, y=15 & 8. a=0.30, b=0.40 \end{array}$$

Exercise 51 (page 96)

$$\begin{array}{ll} 1. x = \frac{1}{2}, y = \frac{1}{4} & 2. a = \frac{1}{3}, b = -\frac{1}{2} \\ 3. p = \frac{1}{4}, q = \frac{1}{5} & 4. x = 10, y = 5 \\ 5. c = 3, d = 4 & 6. r = 3, s = \frac{1}{2} \\ 7. x = 5, y = 1\frac{3}{4} & 8. 1 \end{array}$$

Exercise 52 (page 99)

$$\begin{array}{ll} 1. a=0.2, b=4 & 2. I_1=6.47, I_2=4.62 \\ 3. u=12, a=4, v=26 & 4. £15 500, £12 800 \\ 5. m=-0.5, c=3 & \\ 6. \alpha=0.00426, R_0=22.56 \Omega & 7. a=12, b=0.40 \\ 8. a=4, b=10 & 9. F_1=1.5, F_2=-4.5 \end{array}$$

Exercise 53 (page 100)

$$\begin{array}{ll} 1. x=2, y=1, z=3 & 2. x=2, y=-2, z=2 \\ 3. x=5, y=-1, z=-2 & 4. x=4, y=0, z=3 \end{array}$$

Exercise 121 (page 284)

1. $11.11 \sin(\omega t + 0.324)$ 2. $8.73 \sin(\omega t - 0.173)$
3. $i = 21.79 \sin(\omega t - 0.639)$
4. $v = 5.695 \sin(\omega t + 0.670)$
5. $x = 14.38 \sin(\omega t + 1.444)$
6. (a) $305.3 \sin(314.2t - 0.233)$ V (b) 50 Hz
7. (a) $10.21 \sin(628.3t + 0.818)$ V (b) 100 Hz
(c) 10 ms
8. (a) $79.83 \sin(300\pi t + 0.352)$ V (b) 150 Hz
(c) 6.667 ms

Chapter 31
Exercise 122 (page 288)

1. (a) continuous (b) continuous (c) discrete
(d) continuous
2. (a) discrete (b) continuous (c) discrete (d) discrete

Exercise 123 (page 292)

1. If one symbol is used to represent 10 vehicles, working correct to the nearest 5 vehicles, gives 3.5, 4.5, 6, 7, 5 and 4 symbols respectively.
2. If one symbol represents 200 components, working correct to the nearest 100 components gives: Mon 8, Tues 11, Wed 12, Thurs 12 and Fri 6.5.
3. 6 equally spaced horizontal rectangles, whose lengths are proportional to 35, 44, 62, 68, 49 and 41, respectively.
4. 5 equally spaced horizontal rectangles, whose lengths are proportional to 1580, 2190, 1840, 2385 and 1280 units, respectively.
5. 6 equally spaced vertical rectangles, whose heights are proportional to 35, 44, 62, 68, 49 and 41 units, respectively.
6. 5 equally spaced vertical rectangles, whose heights are proportional to 1580, 2190, 1840, 2385 and 1280 units, respectively.
7. Three rectangles of equal height, subdivided in the percentages shown in the columns of the question. P increases by 20% at the expense of Q and R .
8. Four rectangles of equal height, subdivided as follows: week 1: 18%, 7%, 35%, 12%, 28%; week 2: 20%, 8%, 32%, 13%, 27%; week 3: 22%, 10%, 29%, 14%, 25%; week 4: 20%, 9%, 27%, 19%, 25%. Little change in centres A and B , a reduction of about 8% in C , an increase of about 7% in D and a reduction of about 3% in E .
9. A circle of any radius, subdivided into sectors having angles of 7.5° , 22.5° , 52.5° , 167.5° and 110° , respectively.

10. A circle of any radius, subdivided into sectors having angles of 107° , 156° , 29° and 68° , respectively.
11. (a) £495 (b) 88 12. (a) £16 450 (b) 138

Exercise 124 (page 297)

1. There is no unique solution, but one solution is:
39.3–39.4 1; 39.5–39.6 5; 39.7–39.8 9;
39.9–40.0 17; 40.1–40.2 15; 40.3–40.4 7;
40.5–40.6 4; 40.7–40.8 2.
2. Rectangles, touching one another, having mid-points of 39.35, 39.55, 39.75, 39.95, ... and heights of 1, 5, 9, 17, ...
3. There is no unique solution, but one solution is:
20.5–20.9 3; 21.0–21.4 10; 21.5–21.9 11;
22.0–22.4 13; 22.5–22.9 9; 23.0–23.4 2.
4. There is no unique solution, but one solution is:
1–10 3; 11–19 7; 20–22 12; 23–25 11;
26–28 10; 29–30 5; 31–38 2.
5. 20.95 2; 21.45 3; 21.95 24; 22.45 37; 22.95 46; 23.45 48
6. Rectangles, touching one another, having mid-points of 5.5, 15, 21, 24, 27, 33.5 and 43.5. The heights of the rectangles (frequency per unit class range) are 0.3, 0.78, 4, 4.67, 2.33, 0.5 and 0.2.
7. (10.95 2), (11.45 9), (11.95 19), (12.45 31), (12.95 42), (13.45, 50)
8. A graph of cumulative frequency against upper class boundary having co-ordinates given in the answer to problem 7.
9. (a) There is no unique solution, but one solution is:
2.05–2.09 3; 2.10–2.14 10; 2.15–2.19 11;
2.20–2.24 13; 2.25–2.29 9; 2.30–2.34 2.
(b) Rectangles, touching one another, having mid-points of 2.07, 2.12, ... and heights of 3, 10, ...
(c) Using the frequency distribution given in the solution to part (a) gives 2.095 3; 2.145 13; 2.195 24; 2.245 37; 2.295 46; 2.345 48.
(d) A graph of cumulative frequency against upper class boundary having the co-ordinates given in part (c).

Chapter 32
Exercise 125 (page 300)

1. Mean 7.33, median 8, mode 8
2. Mean 27.25, median 27, mode 26

- Elevation, angle of, 191
 Engineering notation, 57
 Equation of a graph, 135
 Equations, 73
 circles, 236
 cubic, 161
 indicial, 115
 linear and quadratic, simultaneously, 110
 quadratic, 102
 simple, 73
 simultaneous, 90
 Equilateral triangle, 171
 Evaluation of formulae, 28
 trigonometric ratios, 185
 Expectation, 306
 Exponential functions, 118
 graphs of, 120
 Expression, 73
 Exterior angle of triangle, 171
 Extrapolation, 133
- Factorial, 119
 Factorization, 69
 to solve quadratic equations, 102
 Factors, 5, 69
 False axes, 142
 Formula, 28
 quadratic, 106
 Formulae, evaluation of, 28
 list of, 356
 transposition of, 83
 Fractions, 9
 addition and subtraction, 10
 multiplication and division, 12
 on calculator, 26
 Frequency, 200, 289
 relative, 289
 Frequency distribution, 293, 296
 polygon, 293, 296
 Frustum, 252
 Full wave rectified waveform, 260
 Functional notation, 313, 315
- Gradient, 134
 of a curve, 314
 Graph drawing rules, 133
 Graphical solution of equations, 155
 cubic, 161
 linear and quadratic, simultaneously, 110
 quadratic, 156
 simultaneous, 155
 Graphs, 130
 exponential functions, 120
 logarithmic functions, 116
 reducing non-linear to linear form, 147
 sine and cosine, 199
 straight lines, 130, 132
 trigonometric functions, 195
 Grid, 130
 reference, 130
 Grouped data, 292
 mean, median and mode, 300
 standard deviation, 302
 Growth and decay, laws of, 125
- Half-wave rectified waveform, 260
 Hemisphere, 249
 Heptagon, 219
 Hexagon, 219
 Highest common factor (HCF), 5, 51, 66, 69
 Histogram, 293–296, 300
 Hooke's law, 42, 142
 Horizontal bar chart, 289
 component, 269, 283
 Hyperbolic logarithms, 111, 122
 Hypotenuse, 172
- Indices, 47, 64
 laws of, 48, 64
 Indicial equations, 115
 Integers, 1
 Integral calculus, 313
 Integrals, 325
 definite, 328
 standard, 326
 Integration, 313, 325
 of ax^n , 325
 Intercept, y-axis, 135
 Interest, 37
 Interior angles, 165, 171
 Interpolation, 132
 Inverse proportion, 40, 45
 trigonometric function, 185
 Irregular areas, 257
 volumes, 259
 Isosceles triangle, 171
- Lagging angle, 200
 Laws of algebra, 61
 growth and decay, 125
 indices, 48, 64, 316
 logarithms, 113, 150
 precedence, 6, 71
 probability, 307
 Leading angle, 200
 Leibniz notation, 315
 Limiting value, 314
 Linear and quadratic equations
 simultaneously, 110
 graphical solution, 160
 Logarithms, 111
 graphs involving, 116
 laws of, 113, 150
 Long division, 4
 Lower class boundary, 293
 Lowest common multiple (LCM), 5, 10, 75
- Major arc, 231
 sector, 230
 segment, 231
 Maximum value, 156, 199
 Mean, 299, 300
 value of waveform, 260
 Measures of central tendency, 299
 Median, 299
 Member of set, 289
 Mid-ordinate rule, 257
 Minimum value, 156
 Minor arc, 231
 sector, 230
 segment, 230
 Mixed number, 9
 Mode, 299
 Multiple, 5
 Multiplication in algebra, 62
 law of probability, 307
 of fractions, 12
 of number, 3, 19
 Table, 3
- Napierian logarithms, 111, 122
 Natural logarithms, 111, 122
 Newton, 53
 Non right-angled triangles, 205
 Non-terminating decimals, 18
 Nose-to-tail method, 267
 Numerator, 9
- Obtuse angle, 165
 Obtuse-angled triangle, 171
 Octagon, 219
 Ogive, 293, 297
 Ohm's law, 42
 Order of precedence, 6, 13, 71
 with fractions, 13
 with numbers, 6
 Origin, 131