Thus, **45 p as a ratio of £7.65 is 1 : 17**
45 : 765, 9 : 153, 3 : 51 and 1 : 17 are **equivalent ratios** and **1 : 17 is the simplest ratio**.

Problem 7. A glass contains 30 ml of whisky which is 40% alcohol. If 45 ml of water is added and the mixture stirred, what is now the alcohol content?

(i) The 30 ml of whisky contains 40% alcohol $= \frac{40}{100} \times 30 = 12$ ml.

(ii) After 45 ml of water is added we have $30 + 45 = 75$ ml of fluid, of which alcohol is 12 ml.

(iii) Fraction of alcohol present $= \frac{12}{75}$

(iv) Percentage of alcohol present $= \frac{12}{75} \times 100\%$
$= \mathbf{16\%}$.

Problem 8. 20 tonnes of a mixture of sand and gravel is 30% sand. How many tonnes of sand must be added to produce a mixture which is 40% gravel?

(i) Amount of sand in 20 tonnes $= 30\%$ of 20 t $= \frac{30}{100} \times 20 = 6$ t.

(ii) If the mixture has 6 t of sand then amount of gravel $= 20 - 6 = 14$ t.

(iii) We want this 14 t of gravel to be 40% of the new mixture. 1% would be $\frac{14}{40}$ t and 100% of the mixture would be $\frac{14}{40} \times 100\,\text{t} = 35$ t.

(iv) If there is 14 t of gravel then amount of sand $= 35 - 14 = 21$ t.

(v) We already have 6 t of sand, so **amount of sand to be added to produce a mixture with 40% gravel** $= 21 - 6 = \mathbf{15\,t}$.

(Note 1 tonne = 1000 kg.)

Now try the following Practice Exercise

Practice Exercise 25 Further ratios (answers on page 342)

1. Express 130 g as a ratio of 1.95 kg.
2. In a laboratory, acid and water are mixed in the ratio 2 : 5. How much acid is needed to make 266 ml of the mixture?
3. A glass contains 30 ml of gin which is 40% alcohol. If 18 ml of water is added and the mixture stirred, determine the new percentage alcoholic content.
4. A wooden beam 4 m long weighs 84 kg. Determine the mass of a similar beam that is 60 cm long.
5. An alloy is made up of metals P and Q in the ratio 3.25 : 1 by mass. How much of P has to be added to 4.4 kg of Q to make the alloy?
6. 15 000 kg of a mixture of sand and gravel is 20% sand. Determine the amount of sand that must be added to produce a mixture with 30% gravel.

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3.5 Direct proportion

Two quantities are in **direct proportion** when they increase or decrease in the **same ratio**. For example, if 12 cans of lager have a mass of 4 kg, then 24 cans of lager will have a mass of 8 kg; i.e., if the quantity of cans doubles then so does the mass. This is direct proportion.

In the previous section we had an example of mixing 1 shovel of cement to 4 shovels of sand; i.e., the ratio of cement to sand was 1 : 4. So, if we have a mix of 10 shovels of cement and 40 shovels of sand and we wanted to double the amount of the mix then we would need to double both the cement and sand, i.e. 20 shovels of cement and 80 shovels of sand. This is another example of direct proportion.

Here are three laws in engineering which involve direct proportion:

(a) **Hooke's law** states that, within the elastic limit of a material, the strain ε produced is directly proportional to the stress σ producing it, i.e. $\varepsilon \propto \sigma$ (note than '$\propto$' means 'is proportional to').

(b) **Charles's law** states that, for a given mass of gas at constant pressure, the volume V is directly proportional to its thermodynamic temperature T, i.e. $V \propto T$.

(c) **Ohm's law** states that the current I flowing through a fixed resistance is directly proportional to the applied voltage V, i.e. $I \propto V$.

Grouped data:

$$\text{mean, } \bar{x} = \frac{\sum fx}{\sum f}$$

$$\text{standard deviation, } \sigma = \sqrt{\left[\frac{\sum\{f(x-\bar{x})^2\}}{\sum f}\right]}$$

Standard derivatives

y or $f(x)$	$\frac{dy}{dx} = \text{or } f'(x)$
ax^n	anx^{n-1}
$\sin ax$	$a\cos ax$
$\cos ax$	$-a\sin ax$
e^{ax}	ae^{ax}
$\ln ax$	$\frac{1}{x}$

Standard integrals

y	$\int y\,dx$
ax^n	$a\frac{x^{n+1}}{n+1} + c$ (except when $n = -1$)
$\cos ax$	$\frac{1}{a}\sin ax + c$
$\sin ax$	$-\frac{1}{a}\cos ax + c$
e^{ax}	$\frac{1}{a}e^{ax} + c$
$\frac{1}{x}$	$\ln x + c$

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Answers to practice exercises

Chapter 1

Exercise 1 (page 2)

1. 19 kg **2.** 16 m **3.** 479 mm
4. −66 **5.** £565 **6.** −225
7. −2136 **8.** −36 121 **9.** £10 7701
10. −4 **11.** 1487 **12.** 5914
13. 189 g **14.** −70872 **15.** $15 333
16. 89.25 cm
17. $d = 64$ mm, $A = 136$ mm, $B = 100$ mm

Exercise 2 (page 5)

1. (a) 168 (b) 868 **2.** (a) £1827 (b) £4158
3. (a) 8613 kg (b) 584 kg
4. (a) 351 mm (b) 924 mm
5. (a) 10 304 (b) −4433 **6.** (a) 48 m (b) 89 m
7. (a) 259 (b) 56 **8.** (a) 1648 (b) 1060
9. (a) 8067 (b) 3347 **10.** 18 kg

Exercise 3 (page 6)

1. (a) 4 (b) 24 **2.** (a) 12 (b) 360
3. (a) 10 (b) 350 **4.** (a) 90 (b) 2700
5. (a) 2 (b) 210 **6.** (a) 3 (b) 180
7. (a) 5 (b) 210 **8.** (a) 15 (b) 6300
9. (a) 14 (b) 420 420 **10.** (a) 14 (b) 53 900

Exercise 4 (page 8)

1. 59 **2.** 14 **3.** 88 **4.** 5 **5.** −33
6. 22 **7.** 68 **8.** 5 **9.** 2 **10.** 5
11. −1

Chapter 2

Exercise 5 (page 11)

1. $2\frac{1}{7}$ **2.** $7\frac{2}{5}$ **3.** $\frac{22}{7}$ **4.** $\frac{71}{8}$
5. $\frac{4}{11}$ **6.** $\frac{3}{7}, \frac{4}{9}, \frac{1}{2}, \frac{3}{5}, \frac{5}{8}$ **7.** $\frac{8}{25}$ **8.** $\frac{11}{15}$
9. $\frac{9}{30}$ **10.** $\frac{9}{10}$ **11.** $\frac{3}{16}$ **12.** $\frac{3}{16}$
13. $\frac{43}{7}$ **14.** $1\frac{1}{15}$ **15.** $\frac{4}{27}$ **16.** $8\frac{51}{52}$
17. $1\frac{9}{40}$ **18.** $1\frac{16}{21}$ **19.** $\frac{17}{60}$ **20.** $\frac{17}{20}$

Exercise 6 (page 13)

1. $\frac{8}{35}$ **2.** $2\frac{2}{9}$ **3.** $\frac{6}{11}$ **4.** $\frac{5}{12}$ **5.** $\frac{3}{28}$
6. $\frac{3}{5}$ **7.** 11 **8.** $\frac{1}{13}$ **9.** $1\frac{1}{2}$ **10.** $\frac{8}{15}$
11. $2\frac{2}{5}$ **12.** $\frac{5}{12}$ **13.** $3\frac{3}{4}$ **14.** $\frac{12}{23}$ **15.** 4
16. $\frac{3}{4}$ **17.** $\frac{1}{9}$ **18.** 13 **19.** 15 **20.** 400 litres
21. (a) £60, P£36, Q£16 **22.** 2880 litres

Exercise 7 (page 14)

1. $2\frac{1}{18}$ **2.** $-\frac{1}{9}$ **3.** $1\frac{1}{6}$ **4.** $4\frac{3}{4}$ **5.** $\frac{13}{20}$
6. $\frac{7}{15}$ **7.** $4\frac{19}{20}$ **8.** 2 **9.** $7\frac{1}{3}$ **10.** $\frac{1}{15}$
11. 4 **12.** $2\frac{17}{20}$

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Chapter 3

Exercise 8 (page 17)

1. $\frac{13}{20}$ **2.** $\frac{9}{250}$ **3.** $\frac{7}{40}$ **4.** $\frac{6}{125}$

5. (a) $\frac{13}{20}$ (b) $\frac{21}{25}$ (c) $\frac{1}{80}$ (d) $\frac{141}{500}$ (e) $\frac{3}{125}$

6. $4\frac{21}{40}$ **7.** $23\frac{11}{25}$ **8.** $10\frac{3}{200}$ **9.** $6\frac{7}{16}$

10. (a) $1\frac{41}{50}$ (b) $4\frac{11}{40}$ (c) $14\frac{1}{8}$ (d) $15\frac{7}{20}$ (e) $16\frac{17}{80}$

11. 0.625 **12.** 6.6875 **13.** 0.21875 **14.** 11.1875

15. 0.28125

Exercise 9 (page 18)

1. 14.18 **2.** 2.785 **3.** 65.38 **4.** 43.27
5. 1.297 **6.** 0.000528

Exercise 10 (page 19)

1. 80.3 **2.** 329.3 **3.** 72.54 **4.** −124.83
5. 295.3 **6.** 18.3 **7.** [illegible] mm

Exercise 11 (page 20)

1. 4.998 **2.** 47.544 **3.** 385.02 **4.** 582.42
5. 456.9 **6.** 434.82 **7.** 626.1 **8.** 1591.6
9. 0.444 **10.** 0.62963 **11.** 1.563 **12.** 53.455
13. 13.84 **14.** 8.69 **15.** (a) 24.81 (b) 24.812
16. (a) 0.00639 (b) 0.0064 **17.** (a) $8.\dot{4}$ (b) $62.\dot{6}$
18. 2400

Chapter 4

Exercise 12 (page 23)

1. 30.797 **2.** 11927 **3.** 13.62 **4.** 53.832
5. 84.42 **6.** 1.0944 **7.** 50.330 **8.** 36.45
9. 10.59 **10.** 12.325

Exercise 13 (page 24)

1. 12.25 **2.** 0.0361 **3.** 46.923 **4.** 1.296×10^{-3}
5. 2.4430 **6.** 2.197 **7.** 30.96 **8.** 0.0549
9. 219.26 **10.** 5.832×10^{-6}

Exercise 14 (page 25)

1. 0.571 **2.** 40 **3.** 0.13459 **4.** 137.9
5. 14.96 **6.** 19.4481 **7.** 515.36×10^{-6}
8. 1.0871 **9.** 15.625×10^{-9} **10.** 52.70

Exercise 15 (page 25)

1. 2.182 **2.** 11.122 **3.** 185.82
4. 0.8307 **5.** 0.1581 **6.** 2.571
7. 5.273 **8.** 1.256 **9.** 0.30366
10. 1.068 **11.** 3.5×10^{6} **12.** 37.5×10^{3}
13. 4.2×10^{-6} **14.** 202.767×10^{-3} **15.** 18.32×10^{6}

Exercise 16 (page 27)

1. 0.4667 **2.** $\frac{13}{14}$ **3.** 4.458 **4.** 2.732
5. $\frac{1}{21}$ **6.** 0.7083 **7.** $-\frac{9}{10}$ **8.** $3\frac{1}{3}$
9. 2.567 **10.** 0.0276

Exercise 17 (page 27)

1. 0.920[illegible] **2.** 0.7314 **3.** 2.9042 **4.** 0.2719
5. 0.4424 **6.** 0.0321 **7.** 0.4232 **8.** 0.1502
9. −0.6992 **10.** 5.8452

Exercise 18 (page 28)

1. 4.995 **2.** 5.782 **3.** 25.72 **4.** 69.42
5. 0.6977 **6.** 52.92 **7.** 591.0 **8.** 17.90
9. 3.520 **10.** 0.3770

Exercise 19 (page 29)

1. $A = 66.59\,\text{cm}^2$ **2.** $C = 52.78\,\text{mm}$ **3.** $R = 37.5$
4. 159 m/s **5.** 0.407 A **6.** 5.02 mm
7. 0.144 J **8.** $628.8\,\text{m}^2$ **9.** 224.5
10. $14\,230\,\text{kg/m}^3$ **11.** 281.1 m/s **12.** $2.526\,\Omega$

Exercise 20 (page 30)

1. £589.27 **2.** 508.1 W **3.** $V = 2.61\,\text{V}$
4. $F = 854.5$ **5.** $I = 3.81\,\text{A}$ **6.** $T = 14.79\,\text{s}$
7. $E = 3.96\,\text{J}$ **8.** $I = 12.77\,\text{A}$ **9.** $s = 17.25\,\text{m}$
10. $A = 7.184\,\text{cm}^2$ **11.** $V = 7.327$
12. (a) 12.53 h (b) 1 h 40 min, 33 m.p.h.
(c) 13.02 h (d) 13.15 h

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2. $a = 40°, b = 82°, c = 66°,$
$d = 75°, e = 30°, f = 75°$

3. DF, DE **4.** 52° **5.** 122.5°

6. $\phi = 51°, x = 161°$

7. 40°, 70°, 70°, 125°, isosceles

8. $a = 18°50', b = 71°10', c = 68°, d = 90°,$
$e = 22°, f = 49°, g = 41°$

9. $a = 103°, b = 55°, c = 77°, d = 125°,$
$e = 55°, f = 22°, g = 103°, h = 77°,$
$i = 103°, j = 77°, k = 81°$

10. 17° **11.** $A = 37°, B = 60°, E = 83°$

Exercise 79 (page 176)

1. (a) congruent *BAC*, *DAC* (SAS)
(b) congruent *FGE*, *JHI* (SSS)
(c) not necessarily congruent
(d) congruent *QRT*, *SRT* (RHS)
(e) congruent *UVW*, *XZY* (ASA)

2. proof

Exercise 80 (page 178)

1. $x = 16.54\,\text{mm}$, $y = 4.28\,\text{mm}$ **2.** 9 cm, 7.79 cm

3. (a) 2.28 cm (b) 4 cm **4.** 3 m

Exercise 81 (page 180)

1–5. Constructions – see similar constructions in worked problems 30 to 33 on pages 179–180.

Chapter 21

Exercise 82 (page 182)

1. 9 cm **2.** 24 m **3.** 9.54 mm
4. 20.81 cm **5.** 7.21 m **6.** 11.18 cm
7. 24.11 mm **8.** $8^2 + 15^2 = 17^2$
9. (a) 27.20 cm each (b) 45° **10.** 20.81 km
11. 3.35 m, 10 cm **12.** 132.7 nautical miles
13. 2.94 mm **14.** 24 mm

Exercise 83 (page 185)

1. $\sin Z = \frac{9}{41}, \cos Z = \frac{40}{41}, \tan X = \frac{40}{9}, \cos X = \frac{9}{41}$

2. $\sin A = \frac{3}{5}, \cos A = \frac{4}{5}, \tan A = \frac{3}{4}, \sin B = \frac{4}{5},$
$\cos B = \frac{3}{5}, \tan B = \frac{4}{3}$

3. $\sin A = \frac{8}{17}, \tan A = \frac{8}{15}$

4. $\sin X = \frac{15}{113}, \cos X = \frac{112}{113}$

5. (a) $\frac{15}{17}$ (b) $\frac{15}{17}$ (c) $\frac{8}{15}$

6. (a) $\sin\theta = \frac{7}{25}$ (b) $\cos\theta = \frac{24}{25}$

7. (a) 9.434 (b) −0.625

Exercise 84 (page 187)

1. 2.7550 **2.** 4.846 **3.** 36.52
4. (a) 0.8660 (b) −0.1010 (c) 0.5[illegible]
5. 42.33° **6.** 15.25° **7.** [illegible].78° **8.** 7°56′
9. 31°22′ **10.** [illegible]54′ **11.** 29.05° **12.** 20°21′
13. [illegible]86 **14.** 1.803

Exercise 85 (page 189)

1. (a) 12.22 (b) 5.619 (c) 14.87 (d) 8.349
(e) 5.595 (f) 5.275

2. (a) $AC = 5.831\,\text{cm}, \angle A = 59.04°, \angle C = 30.96°$
(b) $DE = 6.928\,\text{cm}, \angle D = 30°, \angle F = 60°$
(c) $\angle J = 62°, HJ = 5.634\,\text{cm}, GH = 10.59\,\text{cm}$
(d) $\angle L = 63°, LM = 6.810\,\text{cm}, KM = 13.37\,\text{cm}$
(e) $\angle N = 26°, ON = 9.125\,\text{cm}, NP = 8.201\,\text{cm}$
(f) $\angle S = 49°, RS = 4.346\,\text{cm}, QS = 6.625\,\text{cm}$

3. 6.54 m **4.** 9.40 mm

Exercise 86 (page 192)

1. 36.15 m **2.** 48 m **3.** 249.5 m **4.** 110.1 m
5. 53.0 m **6.** 9.50 m **7.** 107.8 m
8. 9.43 m, 10.56 m **9.** 60 m

Chapter 22

Exercise 87 (page 198)

1. (a) 42.78° and 137.22° (b) 188.53° and 351.47°
2. (a) 29.08° and 330.92° (b) 123.86° and 236.14°
3. (a) 44.21° and 224.21° (b) 113.12° and 293.12°

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