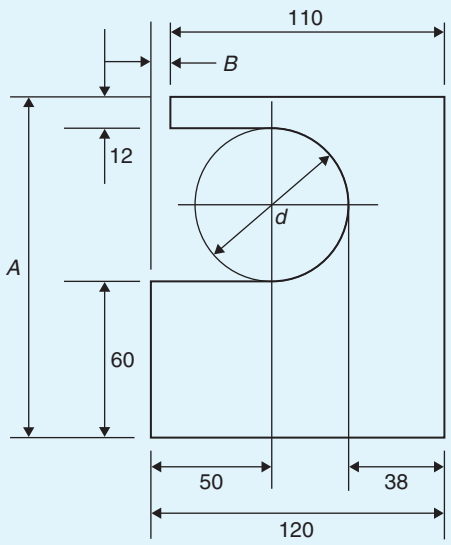

Figure 1.1

Problem 7. Determine 86×7

H	T	U
	8	6
×		7
6	0	2
	4	

(i) $7 \times 6 = 42$. Place the 2 in the units (U) column and 'carry' the 4 into the tens (T) column.

(ii) $7 \times 8 = 56$; $56 + 4$ (carried) $= 60$. Place the 0 in the tens column and the 6 in the hundreds (H) column.

Hence, $86 \times 7 = 602$

A good grasp of **multiplication tables** is needed when multiplying such numbers; a reminder of the multiplication table up to 12×12 is shown below. Confidence with handling numbers will be greatly improved if this table is memorized.

Problem 8. Determine 764×38

$$\begin{array}{r} 764 \\ \times\ 38 \\ \hline 6112 \\ 22920 \\ \hline 29032 \end{array}$$

1.3 Revision of multiplication and division

You can probably already multiply two numbers together and divide one number by another. However, if you need a revision then the following worked problems should be helpful.

Multiplication table

×	2	3	4	5	6	7	8	9	10	11	12
2	4	6	8	10	12	14	16	18	20	22	24
3	6	9	12	15	18	21	24	27	30	33	36
4	8	12	16	20	24	28	32	36	40	44	48
5	10	15	20	25	30	35	40	45	50	55	60
6	12	18	24	30	36	42	48	54	60	66	72
7	14	21	28	35	42	49	56	63	70	77	84
8	16	24	32	40	48	56	64	72	80	88	96
9	18	27	36	45	54	63	72	81	90	99	108
10	20	30	40	50	60	70	80	90	100	110	120
11	22	33	44	55	66	77	88	99	110	121	132
12	24	36	48	60	72	84	96	108	120	132	144

Now try the following Practice Exercise

Practice Exercise 2 Further problems on multiplication and division (answers on page 340)

Determine the values of the expressions given in problems 1 to 9, without using a calculator.

1. (a) 78×6 (b) 124×7
2. (a) £261 × 7 (b) £462 × 9
3. (a) 783 kg × 11 (b) 73 kg × 8
4. (a) 27 mm × 13 (b) 77 mm × 12
5. (a) 448×23 (b) $143 \times (-31)$
6. (a) 288 m ÷ 6 (b) 979 m ÷ 11
7. (a) $\frac{1813}{7}$ (b) $\frac{896}{16}$
8. (a) $\frac{21424}{13}$ (b) 15900 ÷ 15
9. (a) $\frac{88737}{11}$ (b) 46858 ÷ 14
10. A screw has a mass of 15 grams. Calculate, in kilograms, the mass of 1200 such screws (1 kg = 1000 g).

1.4 Highest common factors and lowest common multiples

When two or more numbers are multiplied together, the individual numbers are called **factors**. Thus, a factor is a number which divides into another number exactly. The **highest common factor (HCF)** is the largest number which divides into two or more numbers exactly.

For example, consider the numbers 12 and 15.

The factors of 12 are 1, 2, 3, 4, 6 and 12 (i.e. all the numbers that divide into 12).

The factors of 15 are 1, 3, 5 and 15 (i.e. all the numbers that divide into 15).

1 and 3 are the only **common factors**; i.e., numbers which are factors of **both** 12 and 15.

Hence, **the HCF of 12 and 15 is 3** since 3 is the highest number which divides into **both** 12 and 15.

A **multiple** is a number which contains another number an exact number of times. The smallest number which is exactly divisible by each of two or more numbers is called the **lowest common multiple (LCM)**.

For example, the multiples of 12 are 12, 24, 36, 48, 60, 72,... and the multiples of 15 are 15, 30, 45, 60, 75,...

60 is a common multiple (i.e. a multiple of **both** 12 and 15) and there are no lower common multiples.

Hence, **the LCM of 12 and 15 is 60** since 60 is the lowest number that both 12 and 15 divide into.

Here are some further problems involving the determination of HCFs and LCMs.

Problem 12. Determine the HCF of the numbers 12, 30 and 42

Probably the simplest way of determining an HCF is to express each number in terms of its lowest factors. This is achieved by repeatedly dividing by the prime numbers 2, 3, 5, 7, 11, 13, ... (where possible) in turn. Thus,

$$12 = 2 \times 2 \times 3$$
$$30 = 2 \times 3 \times 5$$
$$42 = 2 \times 3 \times 7$$

The factors which are common to each of the numbers are 2 in column 1 and 3 in column 3, shown by the broken lines. Hence, **the HCF is 2 × 3**; i.e., **6**. That is, 6 is the largest number which will divide into 12, 30 and 42.

Problem 13. Determine the HCF of the numbers 30, 105, 210 and 1155

Using the method shown in Problem 12:

$$30 = 2 \times 3 \times 5$$
$$105 = 3 \times 5 \times 7$$
$$210 = 2 \times 3 \times 5 \times 7$$
$$1155 = 3 \times 5 \times 7 \times 11$$

The factors which are common to each of the numbers are 3 in column 2 and 5 in column 3. Hence, **the HCF is 3 × 5 = 15**.

Problem 14. Determine the LCM of the numbers 12, 42 and 90

Now try the following Practice Exercise

Practice Exercise 4 Further problems on order of precedence and brackets (answers on page 340)

Evaluate the following expressions.

1. $14 + 3 \times 15$
2. $17 - 12 \div 4$
3. $86 + 24 \div (14 - 2)$
4. $7(23 - 18) \div (12 - 5)$
5. $63 - 8(14 \div 2) + 26$
6. $\frac{40}{5} - 42 \div 6 + (3 \times 7)$
7. $\frac{(50 - 14)}{3} + 7(16 - 7) - 7$
8. $\frac{(7 - 3)(1 - 6)}{4(11 - 6) \div (3 - 8)}$
9. $\frac{(3 + 9 \times 6) \div 3 - 2 \div 2}{3 \times 6 + (4 - 9) - 3^2 + 5}$
10. $\frac{\left(4 \times 3^2 + 24\right) \div 5 + 9 \times 3}{2 \times 3^2 - 15 \div 3} + \frac{2 + 27 \div 3 + 12 \div 2 - 3^2}{5 + (13 - 2 \times 5) - 4}$
11. $\frac{1 + \sqrt{25} + 3 \times 2 - 8 \div 2}{3 \times 4 - \sqrt{\left(3^2 + 4^2\right)} + 1} - \frac{(4 \times 2 + 7 \times 2) \div 11}{\sqrt{9} + 14 \div 2 - 2^3}$

Chapter 10

Exercise 39 (page 69)

1. x^2+5x+6 **2.** $2x^2+9x+4$
3. $4x^2+12x+9$ **4.** $2j^2+2j-12$
5. $4x^2+22x+30$ **6.** $2pqr+p^2q^2+r^2$
7. $a^2+2ab+b^2$ **8.** $x^2+12x+36$
9. $a^2-2ac+c^2$ **10.** $25x^2+30x+9$
11. $4x^2-24x+36$ **12.** $4x^2-9$
13. $64x^2+64x+16$ **14.** $r^2s^2+2rst+t^2$
15. $3ab-6a^2$ **16.** $2x^2-2xy$
17. $2a^2-3ab-5b^2$ **18.** $13p-7q$
19. $7x-y-4z$ **20.** $4a^2-25b^2$
21. $x^2-4xy+4y^2$ **22.** $9a^2-6ab+b^2$
23. 0 **24.** $4-a$
25. $4ab-8a^2$ **26.** $3xy+9x^2y-15x^2$
27. $2+5b^2$ **28.** $11q-2p$

Exercise 40 (page 71)

1. $2(x+2)$ **2.** $2x(y-4)$
3. $p(b+2c)$ **4.** $7x(1+2y)$
5. $4d(d-3f^5)$ **6.** $4x(1+2x)$
7. $2y(6-4y)$ **8.** $x(1+3x-5x^2)$
9. $ba(a+b^2)$ **10.** $-(s+p+t)$
11. $3xy(xy^3-5y+6)$ **12.** $2pq^2(2p^2-5q)$
13. $7ab(3ab-4)$ **14.** $2xy(y+3x+4x^2)$
15. $2xy\left(x-2y^2+4x^2y^3\right)$ **16.** $7y(4+y+2x)$
17. $\dfrac{3x}{y}$ **18.** 0 **19.** $\dfrac{2r}{t}$
20. $(a+b)(y+1)$ **21.** $(p+q)(x+y)$
22. $(x-y)(a+b)$ **23.** $(a-2b)(2x+3y)$

Exercise 41 (page 72)

1. $2x+8x^2$ **2.** $12y^2-3y$
3. $4b-15b^2$ **4.** $4+3a$
5. $\dfrac{3}{2}-4x$ **6.** 1
7. $10y^2-3y+\dfrac{1}{4}$ **8.** $9x^2+\dfrac{1}{3}-4x$
9. $6a^2+5a-\dfrac{1}{7}$ **10.** $-15t$
11. $\dfrac{1}{5}-x-x^2$ **12.** $10a^2-3a+2$

Chapter 11

Exercise 42 (page 75)

1. 1 **2.** 2 **3.** 6 **4.** -4 **5.** 2
6. 1 **7.** 2 **8.** $\dfrac{1}{2}$ **9.** 0 **10.** 3
11. 2 **12.** -10 **13.** 6 **14.** -2 **15.** 2.5
16. 2 **17.** 6 **18.** -3

Exercise 43 (page 76)

1. 5 **2.** -2 **3.** $-4\dfrac{1}{2}$ **4.** 2 **5.** 12
6. 15 **7.** -4 **8.** $5\dfrac{1}{3}$ **9.** 2 **10.** 13
11. -10 **12.** 2 **13.** 3 **14.** -11 **15.** -6
16. 9 **17.** $6\dfrac{1}{4}$ **18.** [illegible] **19.** 4 **20.** 10
21. ± 2 **22.** $-3\dfrac{1}{3}$ **23.** ± 3 **24.** ± 4

Exercise 44 (page 79)

1. 10^{-7} **2.** $8\,\text{m/s}^2$ **3.** 3.472
4. (a) $1.8\,\Omega$ (b) $30\,\Omega$
5. digital camera battery £9, camcorder battery £14
6. $800\,\Omega$ **7.** $30\,\text{m/s}^2$

Exercise 45 (page 80)

1. 12 cm, 240 cm^2 **2.** 0.004 **3.** 30
4. 45°C **5.** 50 **6.** £312, £240
7. 30 kg **8.** 12 m, 8 m **9.** 3.5 N

Chapter 12

Exercise 46 (page 84)

1. $d=c-e-a-b$ **2.** $x=\dfrac{y}{7}$
3. $v=\dfrac{c}{p}$ **4.** $a=\dfrac{v-u}{t}$
5. $R=\dfrac{V}{I}$ **6.** $y=\dfrac{1}{3}(t-x)$
7. $r=\dfrac{c}{2\pi}$ **8.** $x=\dfrac{y-c}{m}$

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