

Problem 10. Evaluate 37.6×5.4

$$\begin{array}{r}
 376 \\
 \times 54 \\
 \hline
 1504 \\
 18800 \\
 \hline
 20304
 \end{array}$$

- (i) $376 \times 54 = 20304$.
- (ii) As there are $1 + 1 = 2$ digits to the right of the decimal points of the two numbers being multiplied together, $37.\underline{6} \times 5.\underline{4}$, then

$$37.6 \times 5.4 = 203.04$$

Problem 11. Evaluate $44.25 \div 1.2$, correct to (a) 3 significant figures, (b) 2 decimal places

$$44.25 \div 1.2 = \frac{44.25}{1.2}$$

The denominator is multiplied by 10 to change it into an integer. The numerator is also multiplied by 10 to keep the fraction the same. Thus,

$$\frac{44.25}{1.2} = \frac{44.25 \times 10}{1.2 \times 10} = \frac{442.5}{12}$$

The long division is similar to the long division of integers and the steps are as shown.

$$\begin{array}{r}
 36.875 \\
 12 \overline{) 442.500} \\
 \underline{36} \\
 82 \\
 \underline{72} \\
 105 \\
 \underline{96} \\
 90 \\
 \underline{84} \\
 60 \\
 \underline{60} \\
 0
 \end{array}$$

- (i) 12 into 44 goes 3; place the 3 above the second 4 of 442.500
- (ii) $3 \times 12 = 36$; place the 36 below the 44 of 442.500
- (iii) $44 - 36 = 8$.
- (iv) Bring down the 2 to give 82.
- (v) 12 into 82 goes 6; place the 6 above the 2 of 442.500

- (vi) $6 \times 12 = 72$; place the 72 below the 82.
- (vii) $82 - 72 = 10$.
- (viii) Bring down the 5 to give 105.
- (ix) 12 into 105 goes 8; place the 8 above the 5 of 442.500
- (x) $8 \times 12 = 96$; place the 96 below the 105.
- (xi) $105 - 96 = 9$.
- (xii) Bring down the 0 to give 90.
- (xiii) 12 into 90 goes 7; place the 7 above the first zero of 442.500
- (xiv) $7 \times 12 = 84$; place the 84 below the 90.
- (xv) $90 - 84 = 6$.
- (xvi) Bring down the 0 to give 60.
- (xvii) 12 into 60 gives 5 exactly; place the 5 above the second zero of 442.500
- (xviii) Hence, $44.25 \div 1.2 = \frac{442.5}{12} = 36.875$

So

- (a) $44.25 \div 1.2 = 36.9$, correct to 3 significant figures
- (b) $44.25 \div 1.2 = 36.88$, correct to 2 decimal places.

Problem 12. Express $7\frac{2}{3}$ as a decimal fraction, correct to 4 significant figures

Dividing 2 by 3 gives $\frac{2}{3} = 0.666666\ldots$

and $7\frac{2}{3} = 7.666666\ldots$

Hence, $7\frac{2}{3} = 7.667$ correct to 4 significant figures.

Note that $7.6666\ldots$ is called **7.6 recurring** and is written as $7.\dot{6}$

Now try the following Practice Exercise

Practice Exercise 11 Multiplying and dividing decimal numbers (answers on page 341)

In Problems 1 to 8, evaluate without using a calculator.

- Evaluate 3.57×1.4
- Evaluate 67.92×0.7

List of formulae

Laws of indices:

$$a^m \times a^n = a^{m+n} \quad \frac{a^m}{a^n} = a^{m-n} \quad (a^m)^n = a^{mn}$$

$$a^{m/n} = \sqrt[n]{a^m} \quad a^{-n} = \frac{1}{a^n} \quad a^0 = 1$$

Quadratic formula:

If $ax^2 + bx + c = 0$ then $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Equation of a straight line:

$$y = mx + c$$

Definition of a logarithm:

If $y = a^x$ then $x = \log_a y$

Laws of logarithms:

$$\log(A \times B) = \log A + \log B$$

$$\log\left(\frac{A}{B}\right) = \log A - \log B$$

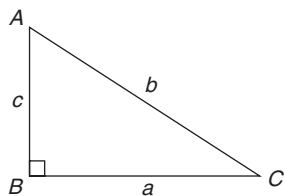
$$\log A^n = n \times \log A$$

Exponential series:

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots \quad (\text{valid for all values of } x)$$

Theorem of Pythagoras:

$$b^2 = a^2 + c^2$$

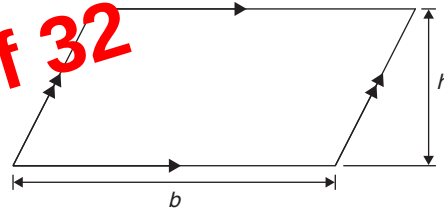


Areas of plane figures:

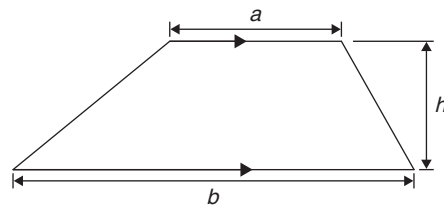
(i) **Rectangle** Area = $l \times b$



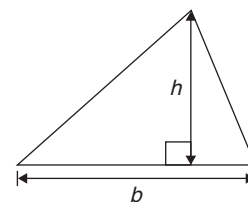
(ii) **Parallelogram** Area = $b \times h$



(iii) **Trapezium** Area = $\frac{1}{2}(a + b)h$



(iv) **Triangle** Area = $\frac{1}{2} \times b \times h$



Chapter 16

Exercise 62 (page 118)

1. (a) 0.1653 (b) 0.4584 (c) 22030
2. (a) 5.0988 (b) 0.064037 (c) 40.446
3. (a) 4.55848 (b) 2.40444 (c) 8.05124
4. (a) 48.04106 (b) 4.07482 (c) -0.08286
5. 2.739 6. 120.7 m

Exercise 63 (page 120)

1. 2.0601 2. (a) 7.389 (b) 0.7408
3. $1 - 2x^2 - \frac{8}{3}x^3 - 2x^4$
4. $2x^{1/2} + 2x^{5/2} + x^{9/2} + \frac{1}{3}x^{13/2}$
 $+ \frac{1}{12}x^{17/2} + \frac{1}{60}x^{21/2}$

Exercise 64 (page 122)

1. 3.95, 2.05 2. 1.65, -1.30
3. (a) 28 cm^3 (b) 116 min 4. (a) 70°C (b) 5 minutes

Exercise 65 (page 124)

1. (a) 0.55547 (b) 0.91374 (c) 8.8941
2. (a) 2.2293 (b) -0.33154 (c) 0.13087
3. -0.4904 4. -0.5822 5. 2.197 6. 816.2
7. 0.8274 8. 11.02 9. 1.522 10. 1.485
11. 1.962 12. 3 13. 4
14. 147.9 15. 4.901 16. 3.095
17. $t = e^{b+a \ln D} = e^b e^{a \ln D} = e^b e^{\ln D^a}$ i.e. $t = e^b D^a$
18. 500 19. $W = PV \ln \left(\frac{U_2}{U_1} \right)$

Exercise 66 (page 127)

1. (a) 150°C (b) 100.5°C 2. 99.21 kPa
3. (a) 29.32 volts (b) $71.31 \times 10^{-6} \text{ s}$
4. (a) 1.993 m (b) 2.293 m 5. (a) 50°C (b) 55.45 s
6. 30.37 N 7. (a) 3.04 A (b) 1.46 s
8. 2.45 mol/cm^3 9. (a) 7.07 A (b) 0.966 s
10. £2424

Chapter 17

Exercise 67 (page 134)

1. (a) Horizontal axis: 1 cm = 4 V (or 1 cm = 5 V),
 vertical axis: 1 cm = 10 Ω
 (b) Horizontal axis: 1 cm = 5 m, vertical axis:
 1 cm = 0.1 V
 (c) Horizontal axis: 1 cm = 10 N, vertical axis:
 1 cm = 0.2 mm
2. (a) -1 (b) -8 (c) -1.5 (d) 5 3. 14.5
4. (a) -1.1 (b) -1.4
5. The 1010 rev/min reading should be 1070 rev/min;
 (a) 1000 rev/min (b) 167 V

Exercise 68 (page 140)

1. Missing values: -0.75, 0.25, 0.75, 2.25, 2.75;
 Gradient = $\frac{1}{2}$
2. (a) 4, -2 (b) -1, 0 (c) -3, -4 (d) 0, 4
3. (a) $2, \frac{1}{2}$ (b) $3, -\frac{2}{2}$ (c) $\frac{1}{24}, \frac{1}{2}$
4. (a) 5, -3 (b) -2, 4 (c) 3, 0 (d) 0, 7
5. (a) $2, -\frac{1}{2}$ (b) $-\frac{2}{3}, -1\frac{2}{3}$ (c) $\frac{1}{18}, 2$ (d) $10, -4\frac{2}{3}$
6. (a) $\frac{3}{5}$ (b) -4 (c) $-1\frac{5}{6}$
7. (a) and (c), (b) and (e)
8. (2, 1) 9. (1.5, 6) 10. (1, 2)
11. (a) 89 cm (b) 11 N (c) 2.4 (d) $1 = 2.4 W + 48$
12. $P = 0.15 W + 3.5$ 13. $a = -20, b = 412$

Exercise 69 (page 144)

1. (a) 40°C (b) 128 Ω
2. (a) 850 rev/min (b) 77.5 V
3. (a) 0.25 (b) 12 (c) $F = 0.25L + 12$
 (d) 89.5 N (e) 592 N (f) 212 N
4. -0.003, 8.73
5. (a) 22.5 m/s (b) 6.43 s (c) $v = 0.7t + 15.5$
6. $m = 26.9L - 0.63$
7. (a) 1.31 t (b) 22.89% (c) $F = -0.09 W + 2.21$
8. (a) $96 \times 10^9 \text{ Pa}$ (b) 0.00022 (c) $28.8 \times 10^6 \text{ Pa}$

9. (a) $\frac{1}{5}$ (b) 6 (c) $E = \frac{1}{5}L + 6$ (d) 12 N (e) 65 N

10. $a = 0.85$, $b = 12$, 254.3 kPa, 275.5 kPa, 280 K

Chapter 18

Exercise 70 (page 149)

1. (a) y (b) x^2 (c) c (d) d 2. (a) y (b) $\sqrt{x}$ (c) b (d) a

3. (a) y (b) $\frac{1}{x}$ (c) f (d) e 4. (a) $\frac{y}{x}$ (b) x (c) b (d) c

5. (a) $\frac{y}{x}$ (b) $\frac{1}{x^2}$ (c) a (d) b

6. $a = 1.5$, $b = 0.4$, 11.78 mm² 7. $y = 2x^2 + 7$, 5.15

8. (a) 950 (b) 317 kN

9. $a = 0.4$, $b = 8.6$ (i) 94.4 (ii) 11.2

Exercise 71 (page 154)

1. (a) $\lg y$ (b) x (c) $\lg a$ (d) $\lg b$

2. (a) $\lg y$ (b) $\lg x$ (c) L (d) $\lg k$

3. (a) $\ln y$ (b) x (c) n (d) $\ln m$

4. $I = 0.0012 \text{ V}^2$, 6.75 cond/m

5. $a = 3.0$, $b = 0.5$

6. $a = 2.7$, $b = 2.6$, 38.53, 3.0

7. $R_0 = 26.0$, $c = 1.42$ 8. $y = 0.08e^{0.24x}$

9. $T_0 = 35.4 \text{ N}$, $\mu = 0.27$, 65.0 N, 1.28 radians

Chapter 19

Exercise 72 (page 156)

1. $x = 2$, $y = 4$ 2. $x = 1$, $y = 1$
 3. $x = 3.5$, $y = 1.5$ 4. $x = -1$, $y = 2$
 5. $x = 2.3$, $y = -1.2$ 6. $x = -2$, $y = -3$
 7. $a = 0.4$, $b = 1.6$

Exercise 73 (page 160)

1. (a) Minimum (0, 0) (b) Minimum (0, -1)
 (c) Maximum (0, 3) (d) Maximum (0, -1)
 2. -0.4 or 0.6 3. -3.9 or 6.9
 4. -1.1 or 4.1 5. -1.8 or 2.2
 6. $x = -1.5$ or -2 , Minimum at $(-1.75, -0.1)$
 7. $x = -0.7$ or 1.6 8. (a) ± 1.63 (b) 1 or -0.3

9. $(-2.6, 13.2)$, $(0.6, 0.8)$; $x = -2.6$ or 0.6

10. $x = -1.2$ or 2.5 (a) -30 (b) 2.75 and -1.50
 (c) 2.3 or -0.8

Exercise 74 (page 161)

1. $x = 4$, $y = 8$ and $x = -0.5$, $y = -5.5$

2. (a) $x = -1.5$ or 3.5 (b) $x = -1.24$ or 3.24
 (c) $x = -1.5$ or 3.0

Exercise 75 (page 162)

1. $x = -2.0$, -0.5 or 1.5

2. $x = -2$, 1 or 3, Minimum at $(2.1, -4.1)$,
 Maximum at $(-0.8, 8.2)$

3. $x = 1$ 4. $x = -2.0$, 0.4 or 2.6

5. $x = 0.7$ or 2.5

6. $x = -2.3$, 1.0 or 1.8 7. $x = 1.5$

Exercise 76 (page 167)

1. 22° 2. $27^\circ 54'$ 3. $51^\circ 11'$ 4. $100^\circ 6' 52''$
 5. $15^\circ 44' 17''$ 6. $86^\circ 49' 1''$ 7. 72.55° 8. 27.754°
 9. $37^\circ 57'$ 10. $58^\circ 22' 52''$

Exercise 77 (page 169)

1. reflex 2. obtuse 3. acute 4. right angle

5. (a) 21° (b) $62^\circ 23'$ (c) $48^\circ 56' 17''$

6. (a) 102° (b) 165° (c) $10^\circ 18' 49''$

7. (a) 60° (b) 110° (c) 75° (d) 143° (e) 140°
 (f) 20° (g) 129.3° (h) 79° (i) 54°

8. Transversal (a) 1 & 3, 2 & 4, 5 & 7, 6 & 8
 (b) 1 & 2, 2 & 3, 3 & 4, 4 & 1, 5 & 6, 6 & 7,
 7 & 8, 8 & 5, 3 & 8, 1 & 6, 4 & 7 or 2 & 5
 (c) 1 & 5, 2 & 6, 4 & 8, 3 & 7 (d) 3 & 5 or 2 & 8

9. $59^\circ 20'$ 10. $a = 69^\circ$, $b = 21^\circ$, $c = 82^\circ$ 11. 51°

12. 1.326 rad 13. 0.605 rad 14. $40^\circ 55'$

Exercise 78 (page 173)

1. (a) acute-angled scalene triangle
 (b) isosceles triangle (c) right-angled triangle
 (d) obtuse-angled scalene triangle
 (e) equilateral triangle (f) right-angled triangle

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