When $R = 5.4 \times 10^3$, $\alpha = 1.215477\ldots \times 10^{-4}$ and $R_0 = 5 \times 10^3$

$$\theta = \frac{1}{1.215477\ldots \times 10^{-4}} \ln\left(\frac{5.4 \times 10^3}{5 \times 10^3}\right)$$

$$= \frac{10^4}{1.215477\ldots}(7.696104\ldots \times 10^{-2})$$

$$= \mathbf{633\,°C} \text{ correct to the nearest degree.}$$

Problem 19. In an experiment involving Newton's law of cooling, the temperature θ(°C) is given by $\theta = \theta_0 e^{-kt}$. Find the value of constant k when $\theta_0 = 56.6$°C, $\theta = 16.5$°C and $t = 79.0$ seconds

Transposing $\theta = \theta_0 e^{-kt}$ gives $\frac{\theta}{\theta_0} = e^{-kt}$, from which

$$\frac{\theta_0}{\theta} = \frac{1}{e^{-kt}} = e^{kt}$$

Taking Napierian logarithms of both sides gives

$$\ln\frac{\theta_0}{\theta} = kt$$

from which,

$$k = \frac{1}{t}\ln\frac{\theta_0}{\theta} = \frac{1}{79.0}\ln\left(\frac{56.6}{16.5}\right)$$

$$= \frac{1}{79.0}(1.2326486\ldots)$$

Hence, $\boldsymbol{k = 0.01560}$ or $\mathbf{15.60 \times 10^{-3}}$.

Problem 20. The current i amperes flowing in a capacitor at time t seconds is given by $i = 8.0(1 - e^{-\frac{t}{CR}})$, where the circuit resistance R is 25 kΩ and capacitance C is 16 μF. Determine (a) the current i after 0.5 seconds and (b) the time, to the nearest millisecond, for the current to reach 6.0 A. Sketch the graph of current against time

(a) Current $i = 8.0\left(1 - e^{-\frac{t}{CR}}\right)$

$$= 8.0[1 - e^{-0.5/(16\times10^{-6})(25\times10^3)}]$$

$$= 8.0(1 - e^{-1.25})$$

$$= 8.0(1 - 0.2865047\ldots)$$

$$= 8.0(0.7134952\ldots)$$

$$= \mathbf{5.71\,amperes}$$

(b) Transposing $i = 8.0\left(1 - e^{-\frac{t}{CR}}\right)$ gives

$$\frac{i}{8.0} = 1 - e^{-\frac{t}{CR}}$$

from which, $e^{-\frac{t}{CR}} = 1 - \frac{i}{8.0} = \frac{8.0 - i}{8.0}$

Taking the reciprocal of both sides gives

$$e^{\frac{t}{CR}} = \frac{8.0}{8.0 - i}$$

Taking Napierian logarithms of both sides gives

$$\frac{t}{CR} = \ln\left(\frac{8.0}{8.0 - i}\right)$$

Hence,

$$t = CR\ln\left(\frac{8.0}{8.0 - i}\right)$$

When $i = 6.0$ A,

$$t = (16 \times 10^{-6})(25 \times 10^3)\ln\left(\frac{8.0}{8.0 - 6.0}\right)$$

i.e. $t = \frac{400}{10^3}\ln\left(\frac{8.0}{2.0}\right) = 0.4\ln 4.0$

$$= 0.4(1.3862943\ldots)$$

$$= 0.5545\,\text{s}$$

$$= \mathbf{555\,ms} \text{ correct to the nearest ms.}$$

A graph of current against time is shown in Figure 16.6.

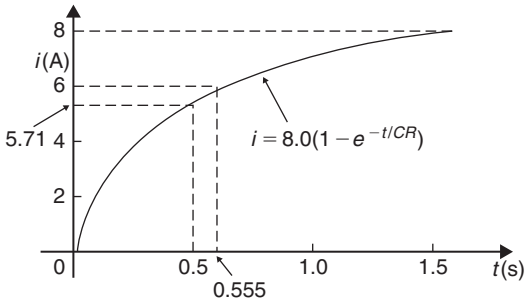


Figure 16.6

Problem 21. The temperature θ_2 of a winding which is being heated electrically at time t is given by $\theta_2 = \theta_1(1 - e^{-\frac{t}{\tau}})$, where θ_1 is the temperature (in degrees Celsius) at time $t = 0$ and τ is a constant. Calculate

List of formulae

Laws of indices:

$$a^m \times a^n = a^{m+n} \quad \frac{a^m}{a^n} = a^{m-n} \quad (a^m)^n = a^{mn}$$

$$a^{m/n} = \sqrt[n]{a^m} \quad a^{-n} = \frac{1}{a^n} \quad a^0 = 1$$

Quadratic formula:

If $ax^2 + bx + c = 0$ then $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Equation of a straight line:

$$y = mx + c$$

Definition of a logarithm:

If $y = a^x$ then $x = \log_a y$

Laws of logarithms:

$$\log(A \times B) = \log A + \log B$$

$$\log\left(\frac{A}{B}\right) = \log A - \log B$$

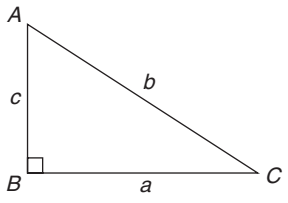
$$\log A^n = n \times \log A$$

Exponential series:

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots \quad \text{(valid for all values of } x\text{)}$$

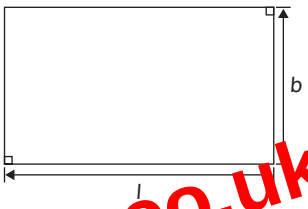
Theorem of Pythagoras:

$$b^2 = a^2 + c^2$$

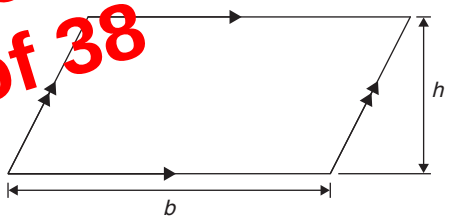


Areas of plane figures:

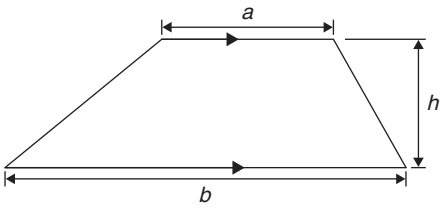
(i) **Rectangle** Area $= l \times b$



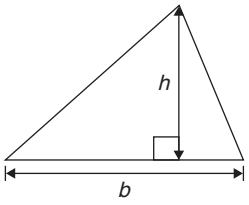
(ii) **Parallelogram** Area $= b \times h$



(iii) **Trapezium** Area $= \frac{1}{2}(a + b)h$



(iv) **Triangle** Area $= \frac{1}{2} \times b \times h$



9. $T = \dfrac{I}{PR}$ **10.** $L = \dfrac{X_L}{2\pi f}$

11. $R = \dfrac{E}{I}$ **12.** $x = a(y-3)$

13. $C = \dfrac{5}{9}(F-32)$ **14.** $f = \dfrac{1}{2\pi CX_C}$

Exercise 47 (page 87)

1. $r = \dfrac{S-a}{S}$ or $1-\dfrac{a}{S}$

2. $x = \dfrac{d}{\lambda}(y+\lambda)$ or $d+\dfrac{yd}{\lambda}$

3. $f = \dfrac{3F-AL}{3}$ or $f = F-\dfrac{AL}{3}$

4. $D = \dfrac{AB^2}{5Ey}$ **5.** $t = \dfrac{R-R_0}{R_0\alpha}$ **6.** $R_2 = \dfrac{RR_1}{R_1-R}$

7. $R = \dfrac{E-e-Ir}{I}$ or $R = \dfrac{E-e}{I} - r$

8. $b = \sqrt{\left(\dfrac{y}{4ac^2}\right)}$ **9.** $x = \dfrac{ay}{\sqrt{(y^2-b^2)}}$

10. $L = \dfrac{t^2 g}{4\pi^2}$ **11.** $u = \sqrt{v^2-2as}$

12. $R = \sqrt{\left(\dfrac{360A}{\pi\theta}\right)}$ **13.** $y = N^2 y - x$

14. $L = \dfrac{\sqrt{Z^2-R^2}}{2\pi f}, 0.080$

Exercise 48 (page 89)

1. $a = \sqrt{\left(\dfrac{xy}{m-n}\right)}$ **2.** $R = \sqrt[4]{\left(\dfrac{M}{\pi}+r^4\right)}$

3. $r = \dfrac{3(x+y)}{(1-x-y)}$ **4.** $L = \dfrac{mrCR}{\mu-m}$

5. $b = \dfrac{c}{\sqrt{1-a^2}}$ **6.** $r = \sqrt{\left(\dfrac{x-y}{x+y}\right)}$

7. $b = \dfrac{a(p^2-q^2)}{2(p^2+q^2)}$ **8.** $v = \dfrac{uf}{u-f}, 30$

9. $t_2 = t_1 + \dfrac{Q}{mc}, 55$ **10.** $v = \sqrt{\left(\dfrac{2dgh}{0.03L}\right)}, 0.965$

11. $L = \dfrac{8S^2}{3d} + d, 2.725$

12. $C = \dfrac{1}{\omega\left\{\omega L - \sqrt{Z^2-R^2}\right\}}, 63.1\times10^{-6}$

13. 64 mm **14.** $\lambda = \sqrt[5]{\left(\dfrac{a\mu}{\rho CZ^4 n}\right)^2}$

Chapter 13

Exercise 49 (page 92)

1. $x=4, y=2$ **2.** $x=3, y=4$

3. $x=2, y=1.5$ **4.** $x=4, y=1$

5. $p=2, q=-1$ **6.** $x=1, y=2$

7. $x=3, y=2$ **8.** $a=2, b=3$

9. $a=5, b=2$ **10.** $x=1, y=1$

11. $s=2, t=3$ **12.** $x=3, y=-2$

13. $m=2.5, n=0.5$ **14.** $a=6, b=-1$

15. $x=2,\ y=5$ **16.** $c=2, d=-3$

Exercise 50 (page 94)

1. $p=-1, q=-2$ **2.** $x=4, y=6$

3. $a=2, b=3$ **4.** $s=4, t=-1$

5. $x=3, y=4$ **6.** $u=12, v=2$

7. $x=10, y=15$ **8.** $a=0.30, b=0.40$

Exercise 51 (page 96)

1. $x=\dfrac{1}{2}, y=\dfrac{1}{4}$ **2.** $a=\dfrac{1}{3}, b=-\dfrac{1}{2}$

3. $p=\dfrac{1}{4}, q=\dfrac{1}{5}$ **4.** $x=10, y=5$

5. $c=3, d=4$ **6.** $r=3, s=\dfrac{1}{2}$

7. $x=5, y=1\dfrac{3}{4}$ **8.** 1

Exercise 52 (page 99)

1. $a=0.2, b=4$ **2.** $I_1=6.47, I_2=4.62$

3. $u=12, a=4, v=26$ **4.** £15 500, £12 800

5. $m=-0.5, c=3$

6. $\alpha=0.00426, R_0=22.56\,\Omega$ **7.** $a=12, b=0.40$

8. $a=4, b=10$ **9.** $F_1=1.5, F_2=-4.5$

Exercise 53 (page 100)

1. $x=2, y=1, z=3$ **2.** $x=2, y=-2, z=2$

3. $x=5, y=-1, z=-2$ **4.** $x=4, y=0, z=3$

Chapter 16

Exercise 62 (page 118)

1. (a) 0.1653 (b) 0.4584 (c) 22030
2. (a) 5.0988 (b) 0.064037 (c) 40.446
3. (a) 4.55848 (b) 2.40444 (c) 8.05124
4. (a) 48.04106 (b) 4.07482 (c) −0.08286
5. 2.739 **6.** 120.7 m

Exercise 63 (page 120)

1. 2.0601 **2.** (a) 7.389 (b) 0.7408
3. $1-2x^2-\frac{8}{3}x^3-2x^4$
4. $2x^{1/2}+2x^{5/2}+x^{9/2}+\frac{1}{3}x^{13/2}+\frac{1}{12}x^{17/2}+\frac{1}{60}x^{21/2}$

Exercise 64 (page 122)

1. 3.95, 2.05 **2.** 1.65, −1.30
3. (a) 28 cm^3 (b) 116 min **4.** (a) 70°C (b) 5 minutes

Exercise 65 (page 124)

1. (a) 0.55547 (b) 0.91374 (c) 8.8941
2. (a) 2.2293 (b) −0.33154 (c) 0.13087
3. −0.4904 **4.** −0.5822 **5.** 2.197 **6.** 816.2
7. 0.8274 **8.** 11.02 **9.** 1.522 **10.** 1.485
11. 1.962 **12.** 3 **13.** 4
14. 147.9 **15.** 4.901 **16.** 3.095
17. $t=e^{b+a\ln D}=e^b e^{a\ln D}=e^b e^{\ln D^a}$ i.e. $t=e^b D^a$
18. 500 **19.** $W=PV\ln\left(\frac{U_2}{U_1}\right)$

Exercise 66 (page 127)

1. (a) 150°C (b) 100.5°C **2.** 99.21 kPa
3. (a) 29.32 volts (b) 71.31×10^{-6} s
4. (a) 1.993 m (b) 2.293 m **5.** (a) 50°C (b) 55.45 s
6. 30.37 N **7.** (a) 3.04 A (b) 1.46 s
8. 2.45 mol/cm^3 **9.** (a) 7.07 A (b) 0.966 s
10. £2424

Chapter 17

Exercise 67 (page 134)

1. (a) Horizontal axis: 1 cm = 4 V (or 1 cm = 5 V), vertical axis: 1 cm = 10 Ω
(b) Horizontal axis: 1 cm = 5 m, vertical axis: 1 cm = 0.1 V
(c) Horizontal axis: 1 cm = 10 N, vertical axis: 1 cm = 0.2 mm
2. (a) −1 (b) −8 (c) −1.5 (d) 5 **3.** 14.5
4. (a) −1.1 (b) −1.4
5. The 1010 rev/min reading should be 1070 rev/min; (a) 1000 rev/min (b) 167 V

Exercise 68 (page 140)

1. Missing values: −0.75, 0.25, 0.75, 2.25, 2.75; Gradient $=\frac{1}{2}$
2. (a) 4, −2 (b) −1, 0 (c) −3, −4 (d) 0, 4
3. (a) 2, $\frac{1}{2}$ (b) 3, $2\frac{1}{2}$ (c) $\frac{1}{24}$, $\frac{1}{2}$
4. (a) 3, −3 (b) −2, 4 (c) 3, 0 (d) 0, 7
5. (a) 2, $-\frac{1}{2}$ (b) $-\frac{2}{3}$, $-1\frac{2}{3}$ (c) $\frac{1}{18}$, 2 (d) 10, $-4\frac{2}{3}$
6. (a) $\frac{3}{5}$ (b) −4 (c) $-1\frac{5}{6}$
7. (a) and (c), (b) and (e)
8. (2, 1) **9.** (1.5, 6) **10.** (1, 2)
11. (a) 89 cm (b) 11 N (c) 2.4 (d) $l=2.4W+48$
12. $P=0.15W+3.5$ **13.** $a=-20, b=412$

Exercise 69 (page 144)

1. (a) 40°C (b) 128 Ω
2. (a) 850 rev/min (b) 77.5 V
3. (a) 0.25 (b) 12 (c) $F=0.25L+12$
(d) 89.5 N (e) 592 N (f) 212 N
4. −0.003, 8.73
5. (a) 22.5 m/s (b) 6.43 s (c) $v=0.7t+15.5$
6. $m=26.9L-0.63$
7. (a) $1.31t$ (b) 22.89% (c) $F=-0.09W+2.21$
8. (a) 96×10^9 Pa (b) 0.00022 (c) 28.8×10^6 Pa

4. $t = 122°7'$ and $237°53'$
5. $\alpha = 218°41'$ and $321°19'$
6. $\theta = 39°44'$ and $219°44'$

Exercise 88 (page 202)

1. 5 **2.** 180° **3.** 30 **4.** 120°
5. 1, 120° **6.** 2, 144° **7.** 3, 90° **8.** 5, 720°
9. $\frac{7}{2}$, 960° **10.** 6, 360° **11.** 4, 180° **12.** 5 ms
13. 40 Hz **14.** 100 μs or 0.1 ms
15. 625 Hz **16.** leading **17.** leading

Exercise 89 (page 203)

1. (a) 40 (b) 25 Hz (c) 0.04 s or 40 ms (c) 25 Hz
(d) 0.29 rad (or 16.62°) leading $40\sin 50\pi t$

2. (a) 75 cm (b) 6.37 Hz (c) 0.157 s
(d) 0.54 rad (or 30.94°) lagging $75\sin 40t$

3. (a) 300 V (b) 100 Hz (c) 0.01 s or 10 ms
(d) 0.412 rad (or 23.61°) lagging $300\sin 200\pi t$

4. (a) $v = 120\sin 100\pi t$ volts
(b) $v = 120\sin(100\pi t + 0.43)$ volts

5. $i = 20\sin\left(80\pi t - \frac{\pi}{6}\right)$ A or
$i = 20\sin(80\pi t - 0.524)$A

6. $3.2\sin(100\pi t + 0.488)$ m

7. (a) 5A, 50 Hz, 20 ms, 24.75° lagging
(b) −2.093A (c) 4.363 A (d) 6.375 ms (e) 3.423 ms

Chapter 23

Exercise 90 (page 207)

1. $C = 83°$, $a = 14.1$ mm, $c = 28.9$ mm,
area $= 189\,\text{mm}^2$

2. $A = 52°2'$, $c = 7.568$ cm, $a = 7.152$ cm,
area $= 25.65\,\text{cm}^2$

3. $D = 19°48'$, $E = 134°12'$, $e = 36.0$ cm,
area $= 134\,\text{cm}^2$

4. $E = 49°0'$, $F = 26°38'$, $f = 15.08$ mm,
area $= 185.6\,\text{mm}^2$

5. $J = 44°29'$, $L = 99°31'$, $l = 5.420$ cm,
area $= 6.132\,\text{cm}^2$, or, $J = 135°31'$, $L = 8°29'$,
$l = 0.811$ cm, area $= 0.917\,\text{cm}^2$

6. $K = 47°8'$, $J = 97°52'$, $j = 62.2$ mm,
area $= 820.2\,\text{mm}^2$ or $K = 132°52'$, $J = 12°8'$,
$j = 13.19$ mm, area $= 174.0\,\text{mm}^2$

Exercise 91 (page 209)

1. $p = 13.2$ cm, $Q = 47.35°$, $R = 78.65°$,
area $= 77.7\,\text{cm}^2$

2. $p = 6.127$ m, $Q = 30.83°$, $R = 44.17°$,
area $= 6.938\,\text{m}^2$

3. $X = 83.33°$, $Y = 52.62°$, $Z = 44.05°$,
area $= 27.8\,\text{cm}^2$

4. $Z = 29.77°$, $Y = 53.50°$, $Z = 96.73°$,
area $= 355\,\text{mm}^2$

Exercise 92 (page 210)

1. 193 km **2.** (a) 122.6 m (b) 94.80°, 40.66°, 44.54°
3. (a) 11.4 m (b) 17.55° **4.** 163.4 m
5. $BF = 3.9$ m, $EB = 4.0$ m **6.** 6.35 m, 5.37 m
7. 32.48 A, 14.31°

Exercise 93 (page 212)

1. 80.42°, 59.38°, 40.20° **2.** (a) 15.23 m (b) 38.07°
3. 40.25 cm, 126.05° **4.** 19.8 cm **5.** 36.2 m
6. $x = 9.3$ mm, $y = 142$ mm **7.** 130° **8.** 13.66 mm

Chapter 24

Exercise 94 (page 215)

1. (5.83, 59.04°) or (5.83, 1.03 rad)
2. (6.61, 20.82°) or (6.61, 0.36 rad)
3. (4.47, 116.57°) or (4.47, 2.03 rad)
4. (6.55, 145.58°) or (6.55, 2.54 rad)
5. (7.62, 203.20°) or (7.62, 3.55 rad)
6. (4.33, 236.31°) or (4.33, 4.12 rad)
7. (5.83, 329.04°) or (5.83, 5.74 rad)
8. (15.68, 307.75°) or (15.68, 5.37 rad)

Exercise 95 (page 217)

1. (1.294, 4.830) **2.** (1.917, 3.960)
3. (−5.362, 4.500) **4.** (−2.884, 2.154)
5. (−9.353, −5.400) **6.** (−2.615, −3.207)
7. (0.750, −1.299) **8.** (4.252, −4.233)
9. (a) $40\angle 18°$, $40\angle 90°$, $40\angle 162°$, $40\angle 234°$, $40\angle 306°$
(b) (38.04, 12.36), (0, 40), (−38.04, 12.36),
(−23.51, −32.36), (23.51, −32.36)
10. 47.0 mm

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