Problem 7. Remove the brackets from the expression and simplify $2[x^2 - 3x(y + x) + 4xy]$

$2[x^2 - 3x(y + x) + 4xy] = 2[x^2 - 3xy - 3x^2 + 4xy]$

(Whenever more than one type of brackets is involved, always **start with the inner brackets**)

$$= 2[-2x^2 + xy]$$
$$= -4x^2 + 2xy$$
$$= \mathbf{2xy - 4x^2}$$

Problem 8. Remove the brackets and simplify the expression $2a - [3\{2(4a - b) - 5(a + 2b)\} + 4a]$

(i) Removing the innermost brackets gives

$$2a - [3\{8a - 2b - 5a - 10b\} + 4a]$$

(ii) Collecting together similar terms gives

$$2a - [3\{3a - 12b\} + 4a]$$

(iii) Removing the 'curly' brackets gives

$$2a - [9a - 36b + 4a]$$

(iv) Collecting together similar terms gives

$$2a - [13a - 36b]$$

(v) Removing the outer brackets gives

$$2a - 13a + 36b$$

(vi) i.e. $\mathbf{-11a + 36b}$ or $\mathbf{36b - 11a}$

Now try the following Practice Exercise

Practice Exercise 39 Brackets (answers on page 344)

Expand the brackets in problems 1 to 28.

1. $(x + 2)(x + 3)$
2. $(x + 4)(2x + 1)$
3. $(2x + 3)^2$
4. $(2j - 4)(j + 3)$
5. $(2x + 6)(2x + 5)$
6. $(pq + r)(r + pq)$
7. $(a + b)(a + b)$
8. $(x + 6)^2$
9. $(a - c)^2$
10. $(5x + 3)^2$
11. $(2x - 6)^2$
12. $(2x - 3)(2x + 3)$
13. $(8x + 4)^2$
14. $(rs + t)^2$
15. $3a(b - 2a)$
16. $2x(x - y)$
17. $(2a - 5b)(a + b)$
18. $3(3p - 2q) - (q - 4p)$
19. $(3x - 4y) + 3(y - z) - (z - 4x)$
20. $(2a + 5b)(2a - 5b)$
21. $(x - 2y)^2$
22. $(3a - b)^2$
23. $2x + [y - (2x + y)]$
24. $3a + 2[a - (3a - 2)]$
25. $4[a^2 - 3a(2b + a) + 7ab]$
26. $3[x^2 - 2x(y + 3x) + 3xy(1 + x)]$
27. $2 - 5[a(a - 2b) - (a - b)^2]$
28. $24p - [2\{3(5p - q) - 2(p + 2q)\} + 3q]$

10.3 Factorization

The **factors** of 8 are 1, 2, 4 and 8 because 8 divides by 1, 2, 4 and 8.

The factors of 24 are 1, 2, 3, 4, 6, 8, 12 and 24 because 24 divides by 1, 2, 3, 4, 6, 8, 12 and 24.

The **common factors** of 8 and 24 are 1, 2, 4 and 8 since 1, 2, 4 and 8 are factors of both 8 and 24.

The **highest common factor (HCF)** is the largest number that divides into two or more terms.

Hence, the HCF of 8 and 24 is 8, as explained in Chapter 1.

When two or more terms in an algebraic expression contain a common factor, then this factor can be shown outside of a bracket. For example,

$$df + dg = d(f + g)$$

which is just the reverse of

$$d(f + g) = df + dg$$

This process is called **factorization**.

Here are some worked examples to help understanding of factorizing in algebra.

Problem 9. Factorize $ab - 5ac$

a is common to both terms ab and $-5ac$. a is therefore taken outside of the bracket. What goes inside the bracket?

Areas of irregular figures by approximate methods:

Trapezoidal rule

$$\text{Area} \approx \begin{pmatrix}\text{width of}\\ \text{interval}\end{pmatrix}\left[\frac{1}{2}\begin{pmatrix}\text{first } + \text{ last}\\ \text{ordinate}\end{pmatrix} + \text{sum of remaining ordinates}\right]$$

Mid-ordinate rule

$$\text{Area} \approx (\text{width of interval})(\text{sum of mid-ordinates})$$

Simpson's rule

$$\text{Area} \approx \frac{1}{3}\begin{pmatrix}\text{width of}\\ \text{interval}\end{pmatrix}\left[\begin{pmatrix}\text{first} + \text{last}\\ \text{ordinate}\end{pmatrix} + 4\begin{pmatrix}\text{sum of even}\\ \text{ordinates}\end{pmatrix} + 2\begin{pmatrix}\text{sum of remaining}\\ \text{odd ordinates}\end{pmatrix}\right]$$

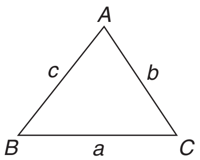
Mean or average value of a waveform:

$$\text{mean value, y} = \frac{\text{area under curve}}{\text{length of base}} = \frac{\text{sum of mid-ordinates}}{\text{number of mid-ordinates}}$$

Triangle formulae:

Sine rule: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

Cosine rule: $a^2 = b^2 + c^2 - 2bc\cos A$



Area of any triangle

$$= \frac{1}{2} \times \text{base} \times \text{perpendicular height}$$

$$= \frac{1}{2}ab\sin C \quad \text{or} \quad \frac{1}{2}ac\sin B \quad \text{or} \quad \frac{1}{2}bc\sin A$$

$$= \sqrt{[s(s-a)(s-b)(s-c)]} \text{ where } s = \frac{a+b+c}{2}$$

For a **general sinusoidal function** $\boldsymbol{y = A\sin(\omega t \pm \alpha)}$, then

A = amplitude

ω = angular velocity $= 2\pi f$ rad/s

$\dfrac{\omega}{2\pi}$ = frequency, f hertz

$\dfrac{2\pi}{\omega}$ = periodic time T seconds

α = angle of lead or lag (compared with $y = A\sin\omega t$)

Cartesian and polar co-ordinates:

If co-ordinate $(x, y) = (r, \theta)$ then

$$r = \sqrt{x^2 + y^2} \quad \text{and} \quad \theta = \tan^{-1}\frac{y}{x}$$

If co-ordinate $(r, \theta) = (x, y)$ then

$$x = r\cos\theta \quad \text{and} \quad y = r\sin\theta$$

Arithmetic progression:

If a = first term and d = common difference, then the arithmetic progression is: $a, a+d, a+2d, \ldots$

The n'th term is: $a + (n-1)d$

Sum of n terms, $S_n = \dfrac{n}{2}[2a + (n-1)d]$

Geometric progression:

If a = first term and r = common ratio, then the geometric progression is: $a, ar, ar^2, \ldots$

The n'th term is: ar^{n-1}

Sum of n terms, $S_n = \dfrac{a(1-r^n)}{(1-r)}$ or $\dfrac{a(r^n-1)}{(r-1)}$

If $-1 < r < 1$, $S_\infty = \dfrac{a}{(1-r)}$

Statistics:

Discrete data:

$$\text{mean, } \bar{x} = \frac{\sum x}{n}$$

$$\text{standard deviation, } \sigma = \sqrt{\left[\frac{\sum(x-\bar{x})^2}{n}\right]}$$

9. (a) $\frac{1}{5}$ (b) 6 (c) $E = \frac{1}{5}L + 6$ (d) 12 N (e) 65 N

10. $a = 0.85$, $b = 12$, 254.3 kPa, 275.5 kPa, 280 K

Chapter 18

Exercise 70 (page 149)

1. (a) y (b) x^2 (c) c (d) d **2.** (a) y (b) $\sqrt{x}$ (c) b (d) a

3. (a) y (b) $\frac{1}{x}$ (c) f (d) e **4.** (a) $\frac{y}{x}$ (b) x (c) b (d) c

5. (a) $\frac{y}{x}$ (b) $\frac{1}{x^2}$ (c) a (d) b

6. $a = 1.5$, $b = 0.4$, 11.78 mm^2 **7.** $y = 2x^2 + 7$, 5.15

8. (a) 950 (b) 317 kN

9. $a = 0.4$, $b = 8.6$ (i) 94.4 (ii) 11.2

Exercise 71 (page 154)

1. (a) $\lg y$ (b) x (c) $\lg a$ (d) $\lg b$

2. (a) $\lg y$ (b) $\lg x$ (c) L (d) $\lg k$

3. (a) $\ln y$ (b) x (c) n (d) $\ln m$

4. $I = 0.0012\,V^2$, 6.75 candelas

5. $a = 3.0$, $b = 0.5$

6. $a = 5.7$, $b = 2.6$, 38.53, 3.0

7. $R_0 = 26.0$, $c = 1.42$ **8.** $y = 0.08e^{0.24x}$

9. $T_0 = 35.4$ N, $\mu = 0.27$, 65.0 N, 1.28 radians

Chapter 19

Exercise 72 (page 156)

1. $x = 2$, $y = 4$ **2.** $x = 1$, $y = 1$

3. $x = 3.5$, $y = 1.5$ **4.** $x = -1$, $y = 2$

5. $x = 2.3$, $y = -1.2$ **6.** $x = -2$, $y = -3$

7. $a = 0.4$, $b = 1.6$

Exercise 73 (page 160)

1. (a) Minimum (0, 0) (b) Minimum (0, −1)
(c) Maximum (0, 3) (d) Maximum (0, −1)

2. −0.4 or 0.6 **3.** −3.9 or 6.9

4. −1.1 or 4.1 **5.** −1.8 or 2.2

6. $x = -1.5$ or −2, Minimum at (−1.75, −0.1)

7. $x = -0.7$ or 1.6 **8.** (a) ±1.63 (b) 1 or −0.3

9. (−2.6, 13.2), (0.6, 0.8); $x = -2.6$ or 0.6

10. $x = -1.2$ or 2.5 (a) −30 (b) 2.75 and −1.50
(c) 2.3 or −0.8

Exercise 74 (page 161)

1. $x = 4$, $y = 8$ and $x = -0.5$, $y = -5.5$

2. (a) $x = -1.5$ or 3.5 (b) $x = -1.24$ or 3.24
(c) $x = -1.5$ or 3.0

Exercise 75 (page 162)

1. $x = -2.0$, −0.5 or 1.5

2. $x = -2$, 1 or 3, Minimum at (2.1, −4.1),
Maximum at (−0.8, 8.2)

3. $x = 1$ **4.** $x = -2.0$, 0.4 or 2.6

5. $x = 0.7$ or 2.5

6. $x = -2.3$, 1.0 or 1.8 **7.** $x = -1.5$

Chapter 20

Exercise 76 (page 167)

1. 32°22′ **2.** 27°54′ **3.** 51°11′ **4.** 100°6′52″

5. 15°44′17″ **6.** 86°49′1″ **7.** 72.55° **8.** 27.754°

9. 37°57′ **10.** 58°22′52″

Exercise 77 (page 169)

1. reflex **2.** obtuse **3.** acute **4.** right angle

5. (a) 21° (b) 62°23′ (c) 48°56′17″

6. (a) 102° (b) 165° (c) 10°18′49″

7. (a) 60° (b) 110° (c) 75° (d) 143° (e) 140°
(f) 20° (g) 129.3° (h) 79° (i) 54°

8. Transversal (a) 1 & 3, 2 & 4, 5 & 7, 6 & 8
(b) 1 & 2, 2 & 3, 3 & 4, 4 & 1, 5 & 6, 6 & 7,
7 & 8, 8 & 5, 3 & 8, 1 & 6, 4 & 7 or 2 & 5
(c) 1 & 5, 2 & 6, 4 & 8, 3 & 7 (d) 3 & 5 or 2 & 8

9. 59°20′ **10.** $a = 69°$, $b = 21°$, $c = 82°$ **11.** 51°

12. 1.326 rad **13.** 0.605 rad **14.** 40°55′

Exercise 78 (page 173)

1. (a) acute-angled scalene triangle
(b) isosceles triangle (c) right-angled triangle
(d) obtuse-angled scalene triangle
(e) equilateral triangle (f) right-angled triangle

Exercise 138 (page 324)

1. $-2542\,\text{A/s}$ **2.** (a) $0.16\,\text{cd/V}$ (b) $312.5\,\text{V}$
3. (a) $-1000\,\text{V/s}$ (b) $-367.9\,\text{V/s}$
4. $-1.635\,\text{Pa/m}$

Chapter 35

Exercise 139 (page 328)

1. (a) $4x+c$ (b) $\dfrac{7x^2}{2}+c$

2. (a) $\dfrac{5}{4}x^4+c$ (b) $\dfrac{3}{8}t^8+c$

3. (a) $\dfrac{2}{15}x^3+c$ (b) $\dfrac{5}{24}x^4+c$

4. (a) $\dfrac{2}{5}x^5-\dfrac{3}{2}x^2+c$ (b) $2t-\dfrac{3}{4}t^4+c$

5. (a) $\dfrac{3x^2}{2}-5x+c$ (b) $4\theta+2\theta^2+\dfrac{\theta^3}{3}+c$

6. (a) $\dfrac{5}{2}\theta^2-2\theta+\theta^3+c$

(b) $\dfrac{3}{4}x^4-\dfrac{2}{3}x^3+\dfrac{3}{2}x^2-2x+c$

7. (a) $-\dfrac{4}{3x}+c$ (b) $-\dfrac{1}{4x^3}+c$

8. (a) $\dfrac{4}{5}\sqrt{x^5}+c$ (b) $\dfrac{1}{9}\sqrt[4]{x^9}+c$

9. (a) $\dfrac{10}{\sqrt{t}}+c$ (b) $\dfrac{15}{7}\sqrt[5]{x}+c$

10. (a) $\dfrac{3}{2}\sin 2x+c$ (b) $-\dfrac{7}{3}\cos 3\theta+c$

11. (a) $-6\cos\dfrac{1}{2}x+c$ (b) $18\sin\dfrac{1}{3}x+c$

12. (a) $\dfrac{3}{8}e^{2x}+c$ (b) $\dfrac{-2}{15e^{5x}}+c$

13. (a) $\dfrac{2}{3}\ln x+c$ (b) $\dfrac{u^2}{2}-\ln u+c$

14. (a) $8\sqrt{x}+8\sqrt{x^3}+\dfrac{18}{5}\sqrt{x^5}+c$

(b) $-\dfrac{1}{t}+4t+\dfrac{4t^3}{3}+c$

Exercise 140 (page 330)

1. (a) 1.5 (b) 0.5 **2.** (a) 105 (b) −0.5
3. (a) 6 (b) −1.333 **4.** (a) −0.75 (b) 0.833
5. (a) 10.67 (b) 0.1667 **6.** (a) 0 (b) 4
7. (a) 1 (b) 4.248 **8.** (a) 0.2352 (b) 2.638
9. (a) 19.09 (b) 2.457 **10.** (a) 0.2703 (b) 9.099

Exercise 141 (page 334)

1. proof **2.** proof **3.** 32 **4.** 29.33 Nm
5. 37.5 **6.** 7.5 **7.** 1
8. 1.67 **9.** 2.67 **10.** 140 m