nearest centimetre, runs from the top of the tent to the peg?

6. In a triangle ABC, $\angle B$ is a right angle, $AB = 6.92$ cm and $BC = 8.78$ cm. Find the length of the hypotenuse.

7. In a triangle CDE, $D = 90°$, $CD = 14.83$ mm and $CE = 28.31$ mm. Determine the length of DE.

8. Show that if a triangle has sides of 8, 15 and 17 cm it is right-angled.

9. Triangle PQR is isosceles, Q being a right angle. If the hypotenuse is 38.46 cm find (a) the lengths of sides PQ and QR and (b) the value of $\angle QPR$.

10. A man cycles 24 km due south and then 20 km due east. Another man, starting at the same time as the first man, cycles 32 km due east and then 7 km due south. Find the distance between the two men.

11. A ladder 3.5 m long is placed against a perpendicular wall with its foot 1.0 m from the wall. How far up the wall (to the nearest centimetre) does the ladder reach? If the foot of the ladder is now moved 30 cm further away from the wall, how far does the top of the ladder fall?

12. Two ships leave a port at the same time. One travels due west at 18.4 knots and the other due south at 27.6 knots. If 1 knot = 1 nautical mile per hour, calculate how far apart the two ships are after 4 hours.

13. Figure 21.7 shows a bolt rounded off at one end. Determine the dimension h.

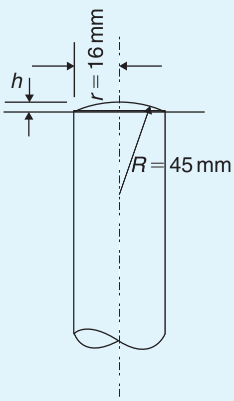


Figure 21.7

14. Figure 21.8 shows a cross-section of a component that is to be made from a round bar. If the diameter of the bar is 74 mm, calculate the dimension x.

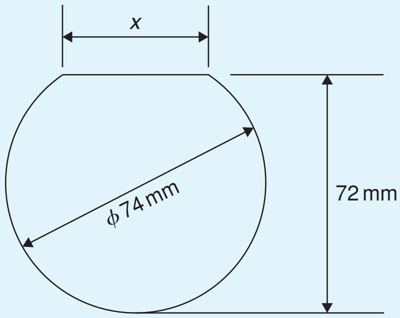


Figure 21.8

21.3 Sines, cosines and tangents

With reference to angle θ in the right-angled triangle ABC shown in Figure 21.9,

$$\textbf{sine}\,\theta = \frac{\textbf{opposite side}}{\textbf{hypotenuse}}$$

'Sine' is abbreviated to 'sin', thus $\mathbf{sin}\,\theta = \dfrac{BC}{AC}$

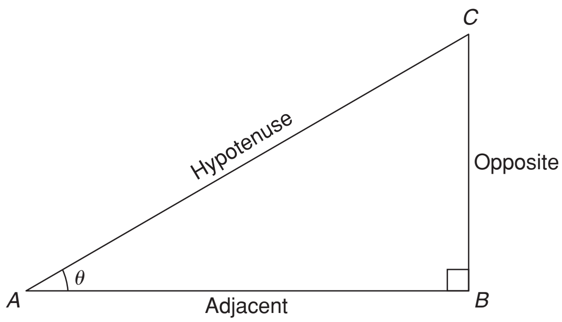


Figure 21.9

Also, $\textbf{cosine}\,\theta = \dfrac{\textbf{adjacent side}}{\textbf{hypotenuse}}$

'Cosine' is abbreviated to 'cos', thus $\mathbf{cos}\,\theta = \dfrac{AB}{AC}$

Finally, $\textbf{tangent}\,\theta = \dfrac{\textbf{opposite side}}{\textbf{adjacent side}}$

'Tangent' is abbreviated to 'tan', thus $\mathbf{tan}\,\theta = \dfrac{BC}{AB}$

These three trigonometric ratios only apply to right-angled triangles. Remembering these three equations is very important and the mnemonic '**SOH CAH TOA**' is one way of remembering them.

SOH indicates **s**in $=$ **o**pposite $\div$ **h**ypotenuse

CAH indicates **c**os $=$ **a**djacent $\div$ **h**ypotenuse

TOA indicates **t**an $=$ **o**pposite $\div$ **a**djacent

Here are some worked problems to help familiarize ourselves with trigonometric ratios.

Problem 4. In triangle PQR shown in Figure 21.10, determine $\sin\theta$, $\cos\theta$ and $\tan\theta$

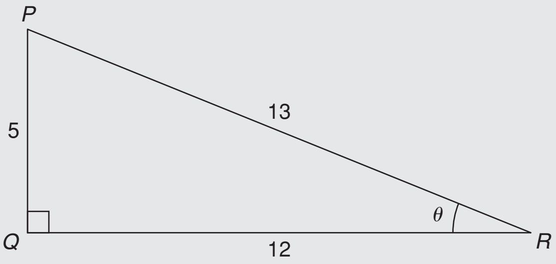


Figure 21.10

$$\sin\theta = \frac{\text{opposite side}}{\text{hypotenuse}} = \frac{PQ}{PR} = \frac{5}{13} = \mathbf{0.3846}$$

$$\cos\theta = \frac{\text{adjacent side}}{\text{hypotenuse}} = \frac{QR}{PR} = \frac{12}{13} = \mathbf{0.9231}$$

$$\tan\theta = \frac{\text{opposite side}}{\text{adjacent side}} = \frac{PQ}{QR} = \frac{5}{12} = \mathbf{0.4167}$$

Problem 5. In triangle ABC of Figure 21.11, determine length AC, $\sin C$, $\cos C$, $\tan C$, $\sin A$, $\cos A$ and $\tan A$

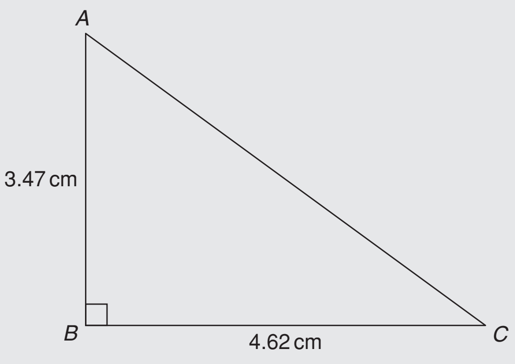


Figure 21.11

By Pythagoras, $AC^2 = AB^2 + BC^2$

i.e. $AC^2 = 3.47^2 + 4.62^2$

from which $AC = \sqrt{3.47^2 + 4.62^2} = \mathbf{5.778\,cm}$

$$\sin C = \frac{\text{opposite side}}{\text{hypotenuse}} = \frac{AB}{AC} = \frac{3.47}{5.778} = \mathbf{0.6006}$$

$$\cos C = \frac{\text{adjacent side}}{\text{hypotenuse}} = \frac{BC}{AC} = \frac{4.62}{5.778} = \mathbf{0.7996}$$

$$\tan C = \frac{\text{opposite side}}{\text{adjacent side}} = \frac{AB}{BC} = \frac{3.47}{4.62} = \mathbf{0.7511}$$

$$\sin A = \frac{\text{opposite side}}{\text{hypotenuse}} = \frac{BC}{AC} = \frac{4.62}{5.778} = \mathbf{0.7996}$$

$$\cos A = \frac{\text{adjacent side}}{\text{hypotenuse}} = \frac{AB}{AC} = \frac{3.47}{5.778} = \mathbf{0.6006}$$

$$\tan A = \frac{\text{opposite side}}{\text{adjacent side}} = \frac{BC}{AB} = \frac{4.62}{3.47} = \mathbf{1.3314}$$

Problem 6. If $\tan B = \frac{8}{15}$, determine the value of $\sin B$, $\cos B$, $\sin A$ and $\tan A$

A right-angled triangle ABC is shown in Figure 21.12. If $\tan B = \frac{8}{15}$, then $AC = 8$ and $BC = 15$.

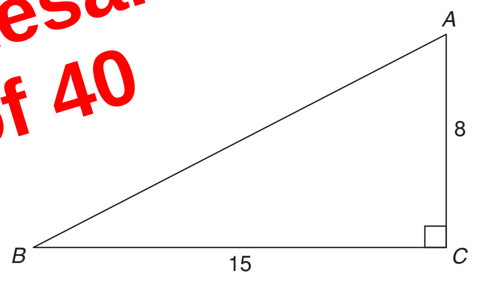


Figure 21.12

By Pythagoras, $AB^2 = AC^2 + BC^2$

i.e. $AB^2 = 8^2 + 15^2$

from which $AB = \sqrt{8^2 + 15^2} = 17$

$$\sin B = \frac{AC}{AB} = \frac{\mathbf{8}}{\mathbf{17}} \text{ or } \mathbf{0.4706}$$

$$\cos B = \frac{BC}{AB} = \frac{\mathbf{15}}{\mathbf{17}} \text{ or } \mathbf{0.8824}$$

$$\sin A = \frac{BC}{AB} = \frac{\mathbf{15}}{\mathbf{17}} \text{ or } \mathbf{0.8824}$$

$$\tan A = \frac{BC}{AC} = \frac{\mathbf{15}}{\mathbf{8}} \text{ or } \mathbf{1.8750}$$

Problem 7. Point A lies at co-ordinate (2, 3) and point B at (8, 7). Determine (a) the distance AB and (b) the gradient of the straight line AB

List of formulae

Laws of indices:

$$a^m \times a^n = a^{m+n} \qquad \frac{a^m}{a^n} = a^{m-n} \qquad (a^m)^n = a^{mn}$$

$$a^{m/n} = \sqrt[n]{a^m} \qquad a^{-n} = \frac{1}{a^n} \qquad a^0 = 1$$

Quadratic formula:

If $ax^2 + bx + c = 0$ then $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Equation of a straight line:

$$y = mx + c$$

Definition of a logarithm:

If $y = a^x$ then $x = \log_a y$

Laws of logarithms:

$$\log(A \times B) = \log A + \log B$$

$$\log\left(\frac{A}{B}\right) = \log A - \log B$$

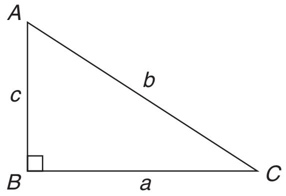
$$\log A^n = n \times \log A$$

Exponential series:

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots \quad \text{(valid for all values of } x\text{)}$$

Theorem of Pythagoras:

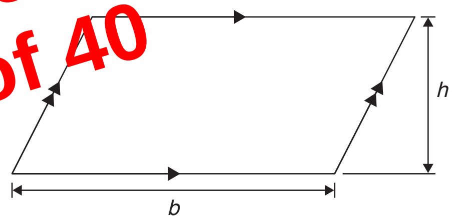
$$b^2 = a^2 + c^2$$



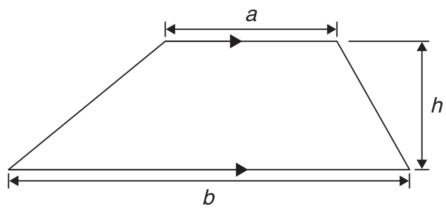
Areas of plane figures:

(i) **Rectangle** Area $= l \times b$

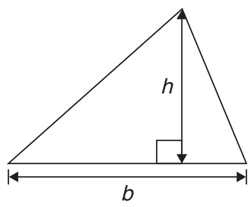
(ii) **Parallelogram** Area $= b \times h$



(iii) **Trapezium** Area $= \frac{1}{2}(a + b)h$



(iv) **Triangle** Area $= \frac{1}{2} \times b \times h$



Areas of irregular figures by approximate methods:

Trapezoidal rule

$$\text{Area} \approx \begin{pmatrix}\text{width of}\\ \text{interval}\end{pmatrix}\left[\frac{1}{2}\begin{pmatrix}\text{first + last}\\ \text{ordinate}\end{pmatrix} + \text{sum of remaining ordinates}\right]$$

Mid-ordinate rule

$$\text{Area} \approx (\text{width of interval})(\text{sum of mid-ordinates})$$

Simpson's rule

$$\text{Area} \approx \frac{1}{3}\begin{pmatrix}\text{width of}\\ \text{interval}\end{pmatrix}\left[\begin{pmatrix}\text{first + last}\\ \text{ordinate}\end{pmatrix} + 4\begin{pmatrix}\text{sum of even}\\ \text{ordinates}\end{pmatrix} + 2\begin{pmatrix}\text{sum of remaining}\\ \text{odd ordinates}\end{pmatrix}\right]$$

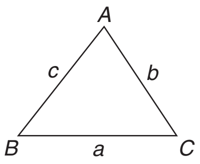
Mean or average value of a waveform:

$$\text{mean value, y} = \frac{\text{area under curve}}{\text{length of base}} = \frac{\text{sum of mid-ordinates}}{\text{number of mid-ordinates}}$$

Triangle formulae:

Sine rule: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

Cosine rule: $a^2 = b^2 + c^2 - 2bc\cos A$



Area of any triangle

$$= \frac{1}{2} \times \text{base} \times \text{perpendicular height}$$

$$= \frac{1}{2}ab\sin C \quad \text{or} \quad \frac{1}{2}ac\sin B \quad \text{or} \quad \frac{1}{2}bc\sin A$$

$$= \sqrt{[s(s-a)(s-b)(s-c)]} \text{ where } s = \frac{a+b+c}{2}$$

For a **general sinusoidal function** $y = A\sin(\omega t \pm \alpha)$, then

$$A = \text{amplitude}$$

$$\omega = \text{angular velocity} = 2\pi f \text{ rad/s}$$

$$\frac{\omega}{2\pi} = \text{frequency, } f \text{ hertz}$$

$$\frac{2\pi}{\omega} = \text{periodic time } T \text{ seconds}$$

$$\alpha = \text{angle of lead or lag (compared with } y = A\sin\omega t)$$

Cartesian and polar co-ordinates:

If co-ordinate $(x, y) = (r, \theta)$ then

$$r = \sqrt{x^2 + y^2} \quad \text{and} \quad \theta = \tan^{-1}\frac{y}{x}$$

If co-ordinate $(r, \theta) = (x, y)$ then

$$x = r\cos\theta \quad \text{and} \quad y = r\sin\theta$$

Arithmetic progression:

If a = first term and d = common difference, then the arithmetic progression is: $a, a+d, a+2d, \ldots$

The n'th term is: $a + (n-1)d$

Sum of n terms, $S_n = \dfrac{n}{2}[2a + (n-1)d]$

Geometric progression:

If a = first term and r = common ratio, then the geometric progression is: $a, ar, ar^2, \ldots$

The n'th term is: ar^{n-1}

Sum of n terms, $S_n = \dfrac{a(1-r^n)}{(1-r)}$ or $\dfrac{a(r^n-1)}{(r-1)}$

If $-1 < r < 1$, $S_\infty = \dfrac{a}{(1-r)}$

Statistics:

Discrete data:

$$\text{mean, } \bar{x} = \frac{\sum x}{n}$$

$$\text{standard deviation, } \sigma = \sqrt{\left[\frac{\sum(x-\bar{x})^2}{n}\right]}$$

Grouped data:

$$\text{mean, } \bar{x} = \frac{\sum fx}{\sum f}$$

$$\text{standard deviation, } \sigma = \sqrt{\left[\frac{\sum\{f(x-\bar{x})^2\}}{\sum f}\right]}$$

Standard derivatives

y or $f(x)$	$\frac{dy}{dx}$ = or $f'(x)$
ax^n	anx^{n-1}
$\sin ax$	$a\cos ax$
$\cos ax$	$-a\sin ax$
e^{ax}	ae^{ax}
$\ln ax$	$\frac{1}{x}$

Standard integrals

y	$\int y\,dx$
ax^n	$a\frac{x^{n+1}}{n+1} + c$ (except when $n = -1$)
$\cos ax$	$\frac{1}{a}\sin ax + c$
$\sin ax$	$-\frac{1}{a}\cos ax + c$
e^{ax}	$\frac{1}{a}e^{ax} + c$
$\frac{1}{x}$	$\ln x + c$

Exercise 138 (page 324)

1. $-2542\,\text{A/s}$ **2.** (a) $0.16\,\text{cd/V}$ (b) $312.5\,\text{V}$
3. (a) $-1000\,\text{V/s}$ (b) $-367.9\,\text{V/s}$
4. $-1.635\,\text{Pa/m}$

Chapter 35

Exercise 139 (page 328)

1. (a) $4x+c$ (b) $\frac{7x^2}{2}+c$

2. (a) $\frac{5}{4}x^4+c$ (b) $\frac{3}{8}t^8+c$

3. (a) $\frac{2}{15}x^3+c$ (b) $\frac{5}{24}x^4+c$

4. (a) $\frac{2}{5}x^5-\frac{3}{2}x^2+c$ (b) $2t-\frac{3}{4}t^4+c$

5. (a) $\frac{3x^2}{2}-5x+c$ (b) $4\theta+2\theta^2+\frac{\theta^3}{3}+c$

6. (a) $\frac{5}{2}\theta^2-2\theta+\theta^3+c$

(b) $\frac{3}{4}x^4-\frac{2}{3}x^3+\frac{3}{2}x^2-2x+c$

7. (a) $-\frac{4}{3x}+c$ (b) $-\frac{1}{4x^3}+c$

8. (a) $\frac{4}{5}\sqrt{x^5}+c$ (b) $\frac{1}{9}\sqrt[4]{x^9}+c$

9. (a) $\frac{10}{\sqrt{t}}+c$ (b) $\frac{15}{7}\sqrt[5]{x}+c$

10. (a) $\frac{3}{2}\sin 2x+c$ (b) $-\frac{7}{3}\cos 3\theta+c$

11. (a) $-6\cos\frac{1}{2}x+c$ (b) $18\sin\frac{1}{3}x+c$

12. (a) $\frac{3}{8}e^{2x}+c$ (b) $\frac{-2}{15e^{5x}}+c$

13. (a) $\frac{2}{3}\ln x+c$ (b) $\frac{u^2}{2}-\ln u+c$

14. (a) $8\sqrt{x}+8\sqrt{x^3}+\frac{18}{5}\sqrt{x^5}+c$

(b) $-\frac{1}{t}+4t+\frac{4t^3}{3}+c$

Exercise 140 (page 330)

1. (a) 1.5 (b) 0.5 **2.** (a) 105 (b) -0.5
3. (a) 6 (b) -1.333 **4.** (a) -0.75 (b) 0.833
5. (a) 10.67 (b) 0.1667 **6.** (a) 0 (b) 4
7. (a) 1 (b) 4.248 **8.** (a) 0.2352 (b) 2.638
9. (a) 19.09 (b) 2.457 **10.** (a) 0.2703 (b) 9.099

Exercise 141 (page 334)

1. proof **2.** proof **3.** 32 **4.** $29.33\,\text{Nm}$
5. 37.5 **6.** 7.5 **7.** 1
8. 1.67 **9.** 2.67 **10.** $140\,\text{m}$