

Chapter 7

Powers, roots and laws of indices

7.1 Introduction

The manipulation of powers and roots is a crucial underlying skill needed in algebra. In this chapter, powers and roots of numbers are explained, together with the laws of indices.

Many worked examples are included to help understanding.

7.2 Powers and roots

7.2.1 Indices

The number 16 is the same as $2 \times 2 \times 2 \times 2$, and $2 \times 2 \times 2 \times 2$ can be abbreviated to 2^4 . When written as 2^4 , 2 is called the **base** and the 4 is called the **index** or **power**. 2^4 is read as ‘two to the power of four’.

Similarly, 3^5 is read as ‘three to the power of 5’.

When the indices are 2 and 3 they are given special names; i.e. 2 is called ‘squared’ and 3 is called ‘cubed’. Thus,

4^2 is called ‘four squared’ rather than ‘4 to the power of 2’ and

5^3 is called ‘five cubed’ rather than ‘5 to the power of 3’.

When no index is shown, the power is 1. For example, 2 means 2^1 .

Problem 1. Evaluate (a) 2^6 (b) 3^4

- (a) 2^6 means $2 \times 2 \times 2 \times 2 \times 2 \times 2$ (i.e. 2 multiplied by itself 6 times), and $2 \times 2 \times 2 \times 2 \times 2 \times 2 = 64$
i.e. $2^6 = 64$

- (b) 3^4 means $3 \times 3 \times 3 \times 3$ (i.e. 3 multiplied by itself 4 times), and $3 \times 3 \times 3 \times 3 = 81$
i.e. $3^4 = 81$

Problem 2. Change the following to index form:

- (a) 32 (b) 625
- (a) (i) To express 32 in its lowest factors, 32 is initially divided by the lowest prime number, i.e. 2.
- (ii) $32 \div 2 = 16$, hence $32 = 2 \times 16$.
- (iii) 16 is also divisible by 2, i.e. $16 = 2 \times 8$. Thus, $32 = 2 \times 2 \times 8$.
- (iv) 8 is also divisible by 2, i.e. $8 = 2 \times 4$. Thus, $32 = 2 \times 2 \times 2 \times 4$.
- (v) 4 is also divisible by 2, i.e. $4 = 2 \times 2$. Thus, $32 = 2 \times 2 \times 2 \times 2 \times 2$.
- (vi) Thus, $32 = 2^5$.
- (b) (i) 625 is not divisible by the lowest prime number, i.e. 2. The next prime number is 3 and 625 is not divisible by 3 either. The next prime number is 5.
- (ii) $625 \div 5 = 125$, i.e. $625 = 5 \times 125$.
- (iii) 125 is also divisible by 5, i.e. $125 = 5 \times 25$. Thus, $625 = 5 \times 5 \times 25$.
- (iv) 25 is also divisible by 5, i.e. $25 = 5 \times 5$. Thus, $625 = 5 \times 5 \times 5 \times 5$.
- (v) Thus, $625 = 5^4$.

Areas of irregular figures by approximate methods:**Trapezoidal rule**

$$\text{Area} \approx \left(\text{width of interval} \right) \left[\frac{1}{2} \left(\text{first + last ordinate} \right) + \text{sum of remaining ordinates} \right]$$

Mid-ordinate rule

$$\text{Area} \approx (\text{width of interval})(\text{sum of mid-ordinates})$$

Simpson's rule

$$\text{Area} \approx \frac{1}{3} \left(\text{width of interval} \right) \left[\left(\text{first + last ordinate} \right) + 4 \left(\text{sum of even ordinates} \right) + 2 \left(\text{sum of remaining odd ordinates} \right) \right]$$

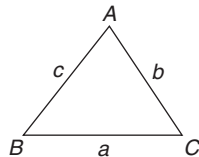
Mean or average value of a waveform:

$$\text{mean value, } y = \frac{\text{area under curve}}{\text{length of base}} = \frac{\text{sum of mid-ordinates}}{\text{number of mid-ordinates}}$$

Triangle formulae:

$$\text{Sine rule: } \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\text{Cosine rule: } a^2 = b^2 + c^2 - 2bc \cos A$$



Area of any triangle

$$= \frac{1}{2} \times \text{base} \times \text{perpendicular height}$$

$$= \frac{1}{2} ab \sin C \quad \text{or} \quad \frac{1}{2} ac \sin B \quad \text{or} \quad \frac{1}{2} bc \sin A$$

$$= \sqrt{[s(s-a)(s-b)(s-c)]} \quad \text{where} \quad s = \frac{a+b+c}{2}$$

For a **general sinusoidal function** $y = A \sin(\omega t \pm \alpha)$, then A = amplitude ω = angular velocity = $2\pi f$ rad/s $\frac{\omega}{2\pi}$ = frequency, f hertz $\frac{2\pi}{\omega}$ = periodic time T seconds α = angle of lead or lag (compared with $y = A \sin \omega t$)**Cartesian and polar co-ordinates:**If co-ordinate $(x, y) = (r, \theta)$ then

$$r = \sqrt{x^2 + y^2} \quad \text{and} \quad \theta = \tan^{-1} \frac{y}{x}$$

If co-ordinate $(r, \theta) = (x, y)$ then

$$x = r \cos \theta \quad \text{and} \quad y = r \sin \theta$$

Arithmetic progression:If a = first term and d = common difference, then the arithmetic progression is: $a, a + d, a + 2d, \dots$ The n 'th term is: $a + (n - 1)d$

$$\text{Sum of } n \text{ terms, } S_n = \frac{n}{2} [2a + (n - 1)d]$$

Geometric progression:If a = first term and r = common ratio, then the geometric progression is: $a, ar, ar^2, \dots$ The n 'th term is: ar^{n-1}

$$\text{Sum of } n \text{ terms, } S_n = \frac{a(1 - r^n)}{(1 - r)} \quad \text{or} \quad \frac{a(r^n - 1)}{(r - 1)}$$

$$\text{If } -1 < r < 1, \quad S_\infty = \frac{a}{(1 - r)}$$

Statistics:

Discrete data:

$$\text{mean, } \bar{x} = \frac{\sum x}{n}$$

$$\text{standard deviation, } \sigma = \sqrt{\left[\frac{\sum (x - \bar{x})^2}{n} \right]}$$

Grouped data:

$$\text{mean, } \bar{x} = \frac{\sum fx}{\sum f}$$

$$\text{standard deviation, } \sigma = \sqrt{\left[\frac{\sum \{f(x - \bar{x})^2\}}{\sum f} \right]}$$

Standard derivatives

y or $f(x)$	$\frac{dy}{dx}$ = or $f'(x)$
ax^n	anx^{n-1}
$\sin ax$	$a \cos ax$
$\cos ax$	$-a \sin ax$
e^{ax}	ae^{ax}
$\ln ax$	$\frac{1}{x}$

Standard integrals

y	$\int y \, dx$
ax^n	$a \frac{x^{n+1}}{n+1} + c$ (except when $n = -1$)
$\cos ax$	$\frac{1}{a} \sin ax + c$
$\sin ax$	$-\frac{1}{a} \cos ax + c$
e^{ax}	$\frac{1}{a} e^{ax} + c$
$\frac{1}{x}$	$\ln x + c$

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Page 10 of 32

Answers

Answers to practice exercises

Chapter 1

Exercise 1 (page 2)

1. 19 kg
2. 16 m
3. 479 mm
4. -66
5. £565
6. -225
7. -2136
8. -36 121
9. £10 7701
10. -4
11. 1487
12. 5914
13. 189 g
14. -70872
15. \$15 333
16. 89.25 cm
17. $d = 64$ mm, $A = 136$ mm, $B = 10$ mm

Exercise 2 (page 5)

1. (a) 468 (b) 868
2. (a) £1827 (b) £4158
3. (a) 8613 kg (b) 584 kg
4. (a) 351 mm (b) 924 mm
5. (a) 10304 (b) -4433
6. (a) 48 m (b) 89 m
7. (a) 259 (b) 56
8. (a) 1648 (b) 1060
9. (a) 8067 (b) 3347
10. 18 kg

Exercise 3 (page 6)

1. (a) 4 (b) 24
2. (a) 12 (b) 360
3. (a) 10 (b) 350
4. (a) 90 (b) 2700
5. (a) 2 (b) 210
6. (a) 3 (b) 180
7. (a) 5 (b) 210
8. (a) 15 (b) 6300
9. (a) 14 (b) 420 420
10. (a) 14 (b) 53 900

Exercise 4 (page 8)

1. 59
2. 14
3. 88
4. 5
5. -33
6. 22
7. 68
8. 5
9. 2
10. 5
11. -1

Chapter 2

Exercise 5 (page 11)

1. $2\frac{1}{7}$
2. $7\frac{2}{5}$
3. $\frac{22}{7}$
4. $\frac{71}{8}$
5. $\frac{4}{11}$
6. $\frac{3}{7}, \frac{4}{2}, \frac{1}{2}, \frac{2}{5}, \frac{5}{8}$
7. $\frac{8}{25}$
8. $\frac{11}{15}$
9. $\frac{9}{30}$
10. $\frac{9}{10}$
11. $\frac{3}{16}$
12. $\frac{3}{16}$
13. $\frac{43}{75}$
14. $\frac{1}{15}$
15. $\frac{4}{27}$
16. $8\frac{51}{52}$
17. $1\frac{9}{40}$
18. $1\frac{16}{21}$
19. $\frac{17}{60}$
20. $\frac{17}{20}$

Exercise 6 (page 13)

1. $\frac{8}{35}$
2. $2\frac{2}{9}$
3. $\frac{6}{11}$
4. $\frac{5}{12}$
5. $\frac{3}{28}$
6. $\frac{3}{5}$
7. 11
8. $\frac{1}{13}$
9. $1\frac{1}{2}$
10. $\frac{8}{15}$
11. $2\frac{2}{5}$
12. $\frac{5}{12}$
13. $3\frac{3}{4}$
14. $\frac{12}{23}$
15. 4
16. $\frac{3}{4}$
17. $\frac{1}{9}$
18. 13
19. 15
20. 400 litres
21. (a) £60, P£36, Q£16
22. 2880 litres

Exercise 7 (page 14)

1. $2\frac{1}{18}$
2. $-\frac{1}{9}$
3. $1\frac{1}{6}$
4. $4\frac{3}{4}$
5. $\frac{13}{20}$
6. $\frac{7}{15}$
7. $4\frac{19}{20}$
8. 2
9. $7\frac{1}{3}$
10. $\frac{1}{15}$
11. 4
12. $2\frac{17}{20}$

Chapter 5

Exercise 21 (page 34)

1. 0.32% 2. 173.4% 3. 5.7% 4. 37.4%
5. 128.5% 6. 0.20 7. 0.0125 8. 0.6875
9. 38.462% 10. (a) 21.2% (b) 79.2% (c) 169%
11. (b), (d), (c), (a) 12. $\frac{13}{20}$ 13. $\frac{5}{16}$ 14. $\frac{9}{16}$
15. $A = \frac{1}{2}$, $B = 50\%$, $C = 0.25$, $D = 25\%$, $E = 0.30$,
 $F = \frac{3}{10}$, $G = 0.60$, $H = 60\%$, $I = 0.85$, $J = \frac{17}{20}$

Exercise 22 (page 36)

1. 21.8 kg 2. 9.72 m
3. (a) 496.4 t (b) 8.657 g (c) 20.73 s 4. 2.25%
5. (a) 14% (b) 15.67% (c) 5.36% 6. 37.8 g
7. 14 minutes 57 seconds 8. 76 g 9. £611
10. 37.49% 11. 39.2% 12. 17% 13. 38.7%
14. 2.7% 15. 5.60 m 16. 3.5%

Exercise 23 (page 38)

1. 2.5% 2. 18% 3. £3.00 4. 1/5000
5. £260 6. £20 000 7. £9176.45 8. £50.25
9. £39.60 10. £917.70 11. £185 000 12. 7.2%
13. A 0.6 kg, B 0.9 kg, C 0.5 kg
14. 54%, 31%, 15%, 0.3 t
15. 20 000 kg (or 20 tonnes)
16. 13.5 mm, 11.5 mm 17. 600 kW

Chapter 6

Exercise 24 (page 41)

1. 36 : 1 2. 3.5 : 1 or 7 : 2 3. 47 : 3
4. 96 cm, 240 cm 5. $5\frac{1}{4}$ hours or 5 hours 15 minutes
6. £3680, £1840, £920 7. 12 cm 8. £2172

Exercise 25 (page 42)

1. 1 : 15 2. 76 ml 3. 25% 4. 12.6 kg
5. 14.3 kg 6. 25 000 kg

Exercise 26 (page 43)

1. £556 2. £66 3. 264 kg 4. 450 g 5. 14.56 kg
6. (a) 0.00025 (b) 48 MPa 7. (a) 440 K (b) 5.76 litre

Exercise 27 (page 45)

1. (a) 2 mA (b) 25 V 2. 170 fr 3. 685.8 mm
4. 83 lb 10 oz 5. (a) 159.1 litres (b) 16.5 gallons
6. 29.4 MPa 7. 584.2 mm 8. \$1012

Exercise 28 (page 46)

1. 3.5 weeks 2. 20 days
3. (a) 9.18 (b) 6.12 (c) 0.3375 4. 50 minutes
5. (a) 300×10^3 (b) 0.375 m^2 (c) $24 \times 10^3 \text{ Pa}$

Exercise 29 (page 48)

1. 17 2. 128 3. 100 000 4. 96 5. 2^4
6. ± 5 7. ± 8 8. 100 9. 1 10. 64

Exercise 30 (page 50)

1. 128 2. 3^9 3. 16 4. $\frac{1}{9}$ 5. 1 6. 8
7. 100 8. 1000 9. $\frac{1}{100}$ or 0.01 10. 5 11. 7^6
12. 3^6 13. 3^6 14. 3^4 15. 1 16. 25
17. $\frac{1}{3^5}$ or $\frac{1}{243}$ 18. 49 19. $\frac{1}{2}$ or 0.5 20. 1

Exercise 31 (page 52)

1. $\frac{1}{3 \times 5^2}$ 2. $\frac{1}{7^3 \times 3^7}$ 3. $\frac{3^2}{2^5}$ 4. $\frac{1}{2^{10} \times 5^2}$
5. 9 6. ± 3 7. $\pm \frac{1}{2}$ 8. $\pm \frac{2}{3}$
9. $\frac{147}{148}$ 10. $-1\frac{19}{56}$ 11. $-3\frac{13}{45}$ 12. $\frac{1}{9}$
13. $-\frac{17}{18}$ 14. 64 15. $4\frac{1}{2}$