Hence, $(2x+3)(2x+1)=0$, from which either $(2x+3)=0$ or $(2x+1)=0$.

Thus, $2x=-3$, from which $\boldsymbol{x=-\frac{3}{2}}$ or $\boldsymbol{-1.5}$

or $2x=-1$, from which $\boldsymbol{x=-\frac{1}{2}}$ or $\boldsymbol{-0.5}$

which may be checked in the original equation.

Problem 11. Solve the quadratic equation $15x^2+2x-8=0$ by factorizing

The factors of $15x^2$ are $15x$ and x or $5x$ and $3x$.
The factors of -8 are -4 are $+2$, or 4 and -2, or -8 and $+1$, or 8 and -1.
By trial and error the only combination that works is

$$\boldsymbol{15x^2+2x-8=(5x+4)(3x-2)}$$

Hence, $(5x+4)(3x-2)=0$, from which either $5x+4=0$ or $3x-2=0$.

Hence, $\boldsymbol{x=-\frac{4}{5}}$ or $\boldsymbol{x=\frac{2}{3}}$

which may be checked in the original equation.

Problem 12. The roots of a quadratic equation are $\frac{1}{3}$ and -2. Determine the equation in x

If the roots of a quadratic equation are, say, α and β, then $(x-\alpha)(x-\beta)=0$.

Hence, if $\alpha=\frac{1}{3}$ and $\beta=-2$,

$$\left(x-\frac{1}{3}\right)(x-(-2))=0$$

$$\left(x-\frac{1}{3}\right)(x+2)=0$$

$$x^2-\frac{1}{3}x+2x-\frac{2}{3}=0$$

$$\boldsymbol{x^2+\frac{5}{3}x-\frac{2}{3}=0}$$

or $\boldsymbol{3x^2+5x-2=0}$

Problem 13. Find the equation in x whose roots are 5 and -5

If 5 and -5 are the roots of a quadratic equation then

$$(x-5)(x+5)=0$$

i.e. $$x^2-5x+5x-25=0$$

i.e. $$\boldsymbol{x^2-25=0}$$

Problem 14. Find the equation in x whose roots are 1.2 and -0.4

If 1.2 and -0.4 are the roots of a quadratic equation then

$$(x-1.2)(x+0.4)=0$$

i.e. $$x^2-1.2x+0.4x-0.48=0$$

i.e. $$\boldsymbol{x^2-0.8x-0.48=0}$$

Now try the following Practice Exercise

Practice Exercise 54 Solving quadratic equations by factorization (answers on page 346)

In problems 1 to 30, solve the given equations by factorization.

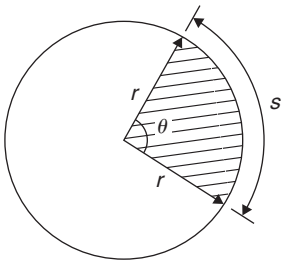
1. $x^2-16=0$
2. $x^2+4x-32=0$
3. $(x+2)^2=16$
4. $4x^2-9=0$
5. $3x^2+4x=0$
6. $8x^2-32=0$
7. $x^2-8x+16=0$
8. $x^2+10x+25=0$
9. $x^2-2x+1=0$
10. $x^2+5x+6=0$
11. $x^2+10x+21=0$
12. $x^2-x-2=0$
13. $y^2-y-12=0$
14. $y^2-9y+14=0$
15. $x^2+8x+16=0$
16. $x^2-4x+4=0$
17. $x^2+6x+9=0$
18. $x^2-9=0$
19. $3x^2+8x+4=0$
20. $4x^2+12x+9=0$
21. $4z^2-\frac{1}{16}=0$
22. $x^2+3x-28=0$
23. $2x^2-x-3=0$
24. $6x^2-5x+1=0$
25. $10x^2+3x-4=0$
26. $21x^2-25x=4$
27. $8x^2+13x-6=0$
28. $5x^2+13x-6=0$
29. $6x^2-5x-4=0$
30. $8x^2+2x-15=0$

In problems 31 to 36, determine the quadratic equations in x whose roots are

31. 3 and 1
32. 2 and -5
33. -1 and -4
34. 2.5 and -0.5
35. 6 and -6
36. 2.4 and -0.7

Preview from Notesale.co.uk
Page 3 of 35

(v) **Circle** Area $= \pi r^2$ Circumference $= 2\pi r$



Radian measure: 2π radians $= 360$ degrees

For a sector of circle:

arc length, $s = \dfrac{\theta^\circ}{360}(2\pi r) = r\theta$ (θ in rad)

shaded area $= \dfrac{\theta^\circ}{360}(\pi r^2) = \dfrac{1}{2}r^2\theta$ (θ in rad)

Equation of a circle, centre at origin, radius r:

$$x^2 + y^2 = r^2$$

Equation of a circle, centre at (a, b), radius r:

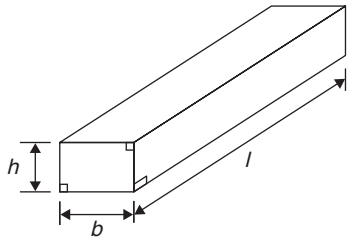
$$(x-a)^2 + (y-b)^2 = r^2$$

Volumes and surface areas of regular solids:

(i) **Rectangular prism (or cuboid)**

Volume $= l \times b \times h$

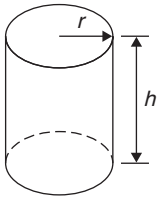
Surface area $= 2(bh + hl + lb)$



(ii) **Cylinder**

Volume $= \pi r^2 h$

Total surface area $= 2\pi rh + 2\pi r^2$

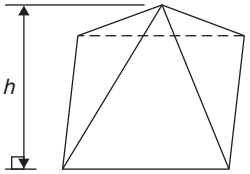


(iii) **Pyramid**

If area of base $= A$ and

perpendicular height $= h$ then:

Volume $= \dfrac{1}{3} \times A \times h$



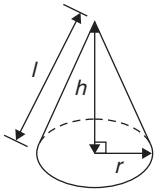
Total surface area = sum of areas of triangles forming sides + area of base

(iv) **Cone**

Volume $= \dfrac{1}{3}\pi r^2 h$

Curved Surface area $= \pi rl$

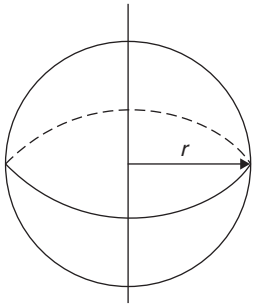
Total Surface area $= \pi rl + \pi r^2$



(v) **Sphere**

Volume $= \dfrac{4}{3}\pi r^3$

Surface area $= 4\pi r^2$



Preview from Notesale.co.uk
Page 11 of 35

9. (a) $\frac{1}{5}$ (b) 6 (c) $E = \frac{1}{5}L + 6$ (d) 12 N (e) 65 N

10. $a = 0.85, b = 12$, 254.3 kPa, 275.5 kPa, 280 K

Chapter 18

Exercise 70 (page 149)

1. (a) y (b) x^2 (c) c (d) d **2.** (a) y (b) $\sqrt{x}$ (c) b (d) a

3. (a) y (b) $\frac{1}{x}$ (c) f (d) e **4.** (a) $\frac{y}{x}$ (b) x (c) b (d) c

5. (a) $\frac{y}{x}$ (b) $\frac{1}{x^2}$ (c) a (d) b

6. $a = 1.5, b = 0.4$, 11.78 mm^2 **7.** $y = 2x^2 + 7$, 5.15

8. (a) 950 (b) 317 kN

9. $a = 0.4, b = 8.6$ (i) 94.4 (ii) 11.2

Exercise 71 (page 154)

1. (a) $\lg y$ (b) x (c) $\lg a$ (d) $\lg b$

2. (a) $\lg y$ (b) $\lg x$ (c) L (d) $\lg k$

3. (a) $\ln y$ (b) x (c) n (d) $\ln m$

4. $I = 0.0012\,V^2$, 6.75 candelas

5. $a = 3.0, b = 0.5$

6. $a = 5.7, b = 2.6$, 38.53, 3.0

7. $R_0 = 26.0, c = 1.42$ **8.** $y = 0.08e^{0.24x}$

9. $T_0 = 35.4$ N, $\mu = 0.27$, 65.0 N, 1.28 radians

Chapter 19

Exercise 72 (page 156)

1. $x = 2, y = 4$ **2.** $x = 1, y = 1$

3. $x = 3.5, y = 1.5$ **4.** $x = -1, y = 2$

5. $x = 2.3, y = -1.2$ **6.** $x = -2, y = -3$

7. $a = 0.4, b = 1.6$

Exercise 73 (page 160)

1. (a) Minimum (0, 0) (b) Minimum (0, −1)
(c) Maximum (0, 3) (d) Maximum (0, −1)

2. −0.4 or 0.6 **3.** −3.9 or 6.9

4. −1.1 or 4.1 **5.** −1.8 or 2.2

6. $x = -1.5$ or −2, Minimum at (−1.75, −0.1)

7. $x = -0.7$ or 1.6 **8.** (a) ± 1.63 (b) 1 or −0.3

9. (−2.6, 13.2), (0.6, 0.8); $x = -2.6$ or 0.6

10. $x = -1.2$ or 2.5 (a) −30 (b) 2.75 and −1.50
(c) 2.3 or −0.8

Exercise 74 (page 161)

1. $x = 4, y = 8$ and $x = -0.5, y = -5.5$

2. (a) $x = -1.5$ or 3.5 (b) $x = -1.24$ or 3.24
(c) $x = -1.5$ or 3.0

Exercise 75 (page 162)

1. $x = -2.0, -0.5$ or 1.5

2. $x = -2$, 1 or 3, Minimum at (2.1, −4.1),
Maximum at (−0.8, 8.2)

3. $x = 1$ **4.** $x = -2.0$, 0.4 or 2.6

5. $x = 0.7$ or 2.5

6. $x = -2.3$, 1.0 or 1.8 **7.** $x = -1.5$

Chapter 20

Exercise 76 (page 167)

1. 22°2′ **2.** 27°54′ **3.** 51°11′ **4.** 100°6′52″

5. 15°44′17″ **6.** 86°49′1″ **7.** 72.55° **8.** 27.754°

9. 37°57′ **10.** 58°22′52″

Exercise 77 (page 169)

1. reflex **2.** obtuse **3.** acute **4.** right angle

5. (a) 21° (b) 62°23′ (c) 48°56′17″

6. (a) 102° (b) 165° (c) 10°18′49″

7. (a) 60° (b) 110° (c) 75° (d) 143° (e) 140°
(f) 20° (g) 129.3° (h) 79° (i) 54°

8. Transversal (a) 1 & 3, 2 & 4, 5 & 7, 6 & 8
(b) 1 & 2, 2 & 3, 3 & 4, 4 & 1, 5 & 6, 6 & 7,
7 & 8, 8 & 5, 3 & 8, 1 & 6, 4 & 7 or 2 & 5
(c) 1 & 5, 2 & 6, 4 & 8, 3 & 7 (d) 3 & 5 or 2 & 8

9. 59°20′ **10.** $a = 69°, b = 21°, c = 82°$ **11.** 51°

12. 1.326 rad **13.** 0.605 rad **14.** 40°55′

Exercise 78 (page 173)

1. (a) acute-angled scalene triangle
(b) isosceles triangle (c) right-angled triangle
(d) obtuse-angled scalene triangle
(e) equilateral triangle (f) right-angled triangle

Preview from Notesale.co.uk
Page 21 of 35

2. $a = 40°, b = 82°, c = 66°,$
$d = 75°, e = 30°, f = 75°$

3. DF, DE **4.** 52° **5.** 122.5°

6. $\phi = 51°, x = 161°$

7. 40°, 70°, 70°, 125°, isosceles

8. $a = 18°50', b = 71°10', c = 68°, d = 90°,$
$e = 22°, f = 49°, g = 41°$

9. $a = 103°, b = 55°, c = 77°, d = 125°,$
$e = 55°, f = 22°, g = 103°, h = 77°,$
$i = 103°, j = 77°, k = 81°$

10. 17° **11.** $A = 37°, B = 60°, E = 83°$

Exercise 79 (page 176)

1. (a) congruent *BAC*, *DAC* (SAS)
(b) congruent *FGE*, *JHI* (SSS)
(c) not necessarily congruent
(d) congruent *QRT*, *SRT* (RHS)
(e) congruent *UVW*, *XZY* (ASA)

2. proof

Exercise 80 (page 178)

1. $x = 16.54\,\text{mm}$, $y = 4.18\,\text{mm}$ **2.** 9 cm, 7.79 cm

3. (a) 2.28 cm (b) 4 cm **4.** 3 m

Exercise 81 (page 180)

1–5. Constructions – see similar constructions in worked problems 30 to 33 on pages 179–180.

Chapter 21

Exercise 82 (page 182)

1. 9 cm **2.** 24 m **3.** 9.54 mm
4. 20.81 cm **5.** 7.21 m **6.** 11.18 cm
7. 24.11 mm **8.** $8^2 + 15^2 = 17^2$
9. (a) 27.20 cm each (b) 45° **10.** 20.81 km
11. 3.35 m, 10 cm **12.** 132.7 nautical miles
13. 2.94 mm **14.** 24 mm

Exercise 83 (page 185)

1. $\sin Z = \frac{9}{41}, \cos Z = \frac{40}{41}, \tan X = \frac{40}{9}, \cos X = \frac{9}{41}$

2. $\sin A = \frac{3}{5}, \cos A = \frac{4}{5}, \tan A = \frac{3}{4}, \sin B = \frac{4}{5},$
$\cos B = \frac{3}{5}, \tan B = \frac{4}{3}$

3. $\sin A = \frac{8}{17}, \tan A = \frac{8}{15}$

4. $\sin X = \frac{15}{113}, \cos X = \frac{112}{113}$

5. (a) $\frac{15}{17}$ (b) $\frac{15}{17}$ (c) $\frac{8}{15}$

6. (a) $\sin\theta = \frac{7}{25}$ (b) $\cos\theta = \frac{24}{25}$

7. (a) 9.434 (b) −0.625

Exercise 84 (page 187)

1. 2.7550 **2.** 4.846 **3.** 36.52
4. (a) 0.8660 (b) −0.1010 (c) 0.5863
5. 42.33° **6.** 15.25° **7.** 7.78° **8.** 7°56′
9. 31°22′ **10.** 1°54′ **11.** 29.05° **12.** 20°21′
13. 1.986 **14.** 1.803

Exercise 85 (page 189)

1. (a) 12.22 (b) 5.619 (c) 14.87 (d) 8.349
(e) 5.595 (f) 5.275

2. (a) $AC = 5.831\,\text{cm}, \angle A = 59.04°, \angle C = 30.96°$
(b) $DE = 6.928\,\text{cm}, \angle D = 30°, \angle F = 60°$
(c) $\angle J = 62°, HJ = 5.634\,\text{cm}, GH = 10.59\,\text{cm}$
(d) $\angle L = 63°, LM = 6.810\,\text{cm}, KM = 13.37\,\text{cm}$
(e) $\angle N = 26°, ON = 9.125\,\text{cm}, NP = 8.201\,\text{cm}$
(f) $\angle S = 49°, RS = 4.346\,\text{cm}, QS = 6.625\,\text{cm}$

3. 6.54 m **4.** 9.40 mm

Exercise 86 (page 192)

1. 36.15 m **2.** 48 m **3.** 249.5 m **4.** 110.1 m
5. 53.0 m **6.** 9.50 m **7.** 107.8 m
8. 9.43 m, 10.56 m **9.** 60 m

Chapter 22

Exercise 87 (page 198)

1. (a) 42.78° and 137.22° (b) 188.53° and 351.47°
2. (a) 29.08° and 330.92° (b) 123.86° and 236.14°
3. (a) 44.21° and 224.21° (b) 113.12° and 293.12°

Preview from Notesale.co.uk
Page 22 of 35

This page intentionally left blank

Preview from Notesale.co.uk
Page 35 of 35