B.Sc. Punjab University 2012

Mathematics B Course **Paper: B**

Time Allowed: 3 Hrs. **Max. Marks: 100**

Attempt SIX questions in all, selecting TWO questions from Section I & II each and ONE question from Section III & IV each.

(Section – I)

1.a) State and prove De Moivre's theorem. 9,8

b) Prove that: $\text{Log}(1 + \cos\theta + i\sin\theta) = \ln\left(2\cos\frac{\theta}{2}\right) + i\frac{\theta}{2}$.

2.a) Evaluate: $1 + x\cos\theta + x^2\cos 2\theta + \ldots + x^n\cos n\theta$. 9,8

b) Prove that in a spherical triangle ABC,

$$\sin\left(\frac{A}{2}\right) = \sqrt{\frac{\sin(s-b)\sin(s-c)}{\sin b\sin c}}$$

3.a) Let $f(x,y) = \begin{cases} \dfrac{x^3+y^3}{x^2+y^2} & \text{if } (x,y) \neq (0,0) \\ 0 & \text{if } (x,y) = (0,0) \end{cases}$ 9,8

Prove that f is not continuous at at (0,0). Do $f_x(0,0)$ and $f_y(0,0)$ exist?

b) Show that the surfaces $z = 16 - x^2 - y^2$ and $63z = x^2 + y^2$ intersect orthogonally.

4.a) Let $U = x^2 + y^2$. Find direction of greatest rate of change of U at (a, b) and magnitude of this greatet rate of change. Find the direction of no change at (a,b) 9,8

b) Find the minimum distance between the lines $x = t, y = 3 - 2t, z = 1 + 2t$ and $x = -1 - s, y = s, z = 4 - 3s$.

(Section – II)

5. a) Determine whether the series $\sum_{1}^{\infty}\frac{\arctan n}{1+n^2}$ converges or diverges. 9,8

b) Find those positive values of x for which the series $1 + \sum_{1}^{\infty}\frac{x^{2n}}{2n}$ converges.

6. a) Prove that if the series $\sum_{1}^{\infty}|a_n|$ converges then so does the series $\sum_{1}^{\infty}a_n$ 9,8

b) Find radius of convergence and interval of convergence of the power series $\sum_{1}^{\infty}\frac{(-1)^{n+1}(x+1)^{2n}}{(n+1)^2 5^n}$

7.a) Find volume of the solid generated by revolving the area bounded by the parabola $y^2 = 8x$ and its latusrectum $(x = 2)$ about the y-axis. 9,8

b) Determine whether the improper integral $\int_0^{\infty}\frac{x^3}{x^3+1}dx$ converges or diverges.

8.a) Find moment of inertia of a spherical ball of constant density with respect to a diameter. 9,8

b) Find the area bounded by $x^2 = 4y$ and $8y = x^2 + 16$

(Section – III)

9. a) Show that every subgroup of a cyclic group is cyclic. 8,8

b) Let H be a subgroup of a group G and $a \in G$. If $(Ha)^{-1} = \{(ha)^{-1} | h \in H|$, then show that $(Ha)^{-1} = a^{-1}H$.

10. a) Prove that for all a, b in a group G, 8,8

$(ab)^{-1} = b^{-1}a^{-1}$

b) Determine whether the permutation $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 4 & 3 & 1 & 2 & 6 & 7 & 5 \end{pmatrix}$ is even or odd.

(Section – IV)

11. a) Define an open set in a metric space. Show that an open sphere in a metric space X is an open set. 8,8

b) Show that closed intervals are closed sets in metric space R.

12. a) If A,B are two subsets of a metric space X; then prove that 8,8

$Ext(A \cup B) = Ext(A) \cap Ext(B)$.

b) Show that the limit of a convergent sequence in a metric space is unique.