A- COURSE OF MATHEMATICS

PAPER: **A**

TIME ALLOWED: 3 hours
MAX. MARKS: 100

Attempt **SIX** questions by selecting TWO questions from Sections - I, TWO from Section – II, ONE from Section - III and ONE from Section - IV

SECTION – I

1. a) Find the values of 'a' and 'b' so that f is continuous and differentiable at x = 1 where $f(x) = \begin{cases} x^3 \text{ if } x < 1 \\ ax + b \text{ if } x \geq 1 \end{cases}$ 9

b) Evaluate $\lim_{x \to 0} x\left[\frac{1}{x}\right]$, $[\cdots]$ being the bracket function.

2. a) Differentiate $\log_{10}^{\left(\frac{x+1}{x}\right)}$ with respect to x. 9

b) Use the Newton – Raphson method to approximate upto four places of decimal, root of $x^3 - 5x + 3 = 0$ with $x_0 = 0$. 8

3. a) Show that $\frac{d^n}{dx^n}\left(\frac{\ln x}{x}\right) = \frac{(-1)^n n!}{x^{n+1}}\left[\ln x - 1 - \frac{1}{2} - \frac{1}{3} \cdots - \frac{1}{n}\right]$ 9

b) If f is a thrice differentiable function, prove by L' Hospital's Rule that 8

$$\lim_{h \to 0} \frac{f(x+h) - f(x) - hf'(x) - \frac{h^2}{2}f''(x)}{h^3} = \frac{f'''(x)}{6}.$$

4. a) If $x > 0$ prove that $x - \ell n(1+x) > \frac{x^2}{2(1+x)}$ 9

b) Prove that under certain conditions (to be stated) 8
$f(a+h) = f(a) + hf'(a + \theta h)$, where $0 < \theta < 1$. Prove also that the limiting value of θ, when h decreases infinitely, is $\frac{1}{2}$

SECTION – II

5. a) If a tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, with centre "C" meets the major and minor axes in T and t, prove that $\frac{a^2}{CT^2} + \frac{b^2}{ct^2} = 1$. 9

b) Show that the curves $r^m = a^m \cos m\theta$ and $r^m = a^m \sin m\theta$ cut each other orthogonally. 8

6. a) Show that the locus of the middle points of a system of parallel chords of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is $y = \frac{-b^2}{a^2 m}x$ where m is slope of the chords. 9

b) Examine whether the equation $2x^2 - xy + 5x - 2y + 2 = 0$ represents two straight lines. If so find equation of each straight line. 8

7. a) Find the equation of the perpendicular from the point P (1, 6, 3) to the straight line $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$. Also obtain its length and coordinates of the foot of the perpendicular. 9

b) Prove that the straight lines $\frac{x-1}{2} = \frac{y+1}{-3} = \frac{z+10}{8}$ and $\frac{x-4}{1} = \frac{y+3}{-4} = \frac{z+1}{7}$ intersect. Also find the point of intersection and the plane through them. 8

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