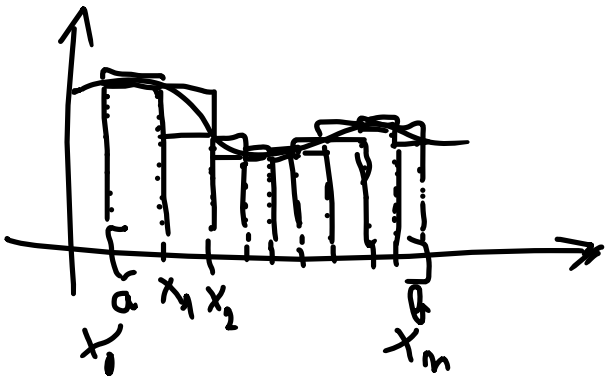
Riemann Integral

Let $f[a,b] \to R$ be bounded



Firstly, let's define what is a partition P of $[a,b]$ is a finite set of points $x_0, \ldots, x_n$ such that:

$$a = x_0 \leq x_1 \leq x_2 \leq \cdots \leq x_{n-1} \leq x_n = b$$

Since we have the partitions defined, now we can show what the Riemann Sum is. There are two Riemann Sums.

The upper Riemann Sum is:

$$U(f,P) = \sum_{i=1}^{n} [sup\ f(x)]_{x \in (x_{i-1}, x_i)} \cdot (x_i - x_{i-1})$$

The lower Riemann Sum is:

$$L(f,P) = \sum_{i=1}^{n} [inf\ f(x)]_{x \in (x_{i-1}, x_i)} \cdot (x_i - x_{i-1})$$

These are in fact, the sums of the areas of the rectangles which are used to approximate the area under a curve. Their difference stands in, that the URS overapproximates the area of under the curve, whereas the LRS underapproximates it.

This happens because of the definition of the supremum and the infimum, which in the formulas above represent the height of the rectangle respectively.