Taylor Series Expansion

First and foremost, Taylor Series is a Power Series!

So, let's start with a reminder of what a Power Series is:

A power series has a radius/interval of convergence.

If $x \in$ interval of convergence around $c, x \in [c-a, c+a], then$

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(c)}{n!} (x-c)^n$$

For this to be useful, c should be close enough to x.

Example: Taylor Series for $f^{(k)}(x) = e^x$, so $f^{(k)}(c) = e^0 = 1$. Thus:

$$\sum_{k=0}^{\infty} \frac{1}{k!} x^k$$

and the radius of the convergence is ∞, i.e., for $x \in R$, we have

$$e^x = \sum_{k=0}^{\infty} \frac{x^k}{k!}$$

Preview from Notesale.co.uk
Page 2 of 6

For a numerical algorithm, we need to stop the summation after a finite number of terms:

Ex.:

$$e^x \approx \frac{1}{0!} + \frac{1}{1!}x + \frac{1}{2!}x^2 + \cdots = 1 + x + \frac{1}{2}x^2$$

Taylor Series of a polynomial is the polynomial itself.