Compute $ln(2)$

We can choose the Taylor series for $ln(x+1)$ and evaluate at $x=1$. In which case:

$$ln(2) = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} + \cdots$$

By keeping 8 terms, we obtain the following result:

$$ln(2) \approx 0.63452$$

Via calculator we get $ln(2) \approx 0.69314$.

In order to get a more accurate approximation, we can use another function's Taylor series. We can try $\ln\left(\frac{1+x}{1-x}\right)$.

From using the logarithm's argument division rule, we have

$$\ln\left(\frac{1+x}{1-x}\right) = \ln(1+x) - \ln(1-x)$$

If we choose $x = \frac{1}{3}$ instead of $x = 1$, we then get

$$\ln(2) = 2\left(\frac{1}{3} + \frac{1}{3^3 \cdot 3} + \frac{1}{3^5 \cdot 5} + \cdots\right)$$

This time, after only 4 terms we have a much better estimate of the result, at:

$$\ln(2) \approx 0.69313$$

Theorem: Reformation of Taylor's Theorem

Assume that $f \in C^{n+1}([a,b])$. We change c to x and the old x becomes $x+h$, where $x, x+h \in [a,b]$.

$$f(x+h) = \sum_{k=0}^{n} \frac{f^{(k)}(x)}{k!} h^k + \frac{f^{(n+1)}(\psi_h)}{(n+1)!} h^{n+1}$$

where

$$\psi_h \in [a,b]$$