This can be rearranged into sigma notation:

$$\sum_{k=N}^{\infty} |a_N| = |a_N| \cdot \underbrace{\sum_{k=0}^{\infty} R^n}_{\text{geometric series}}$$

Since above we defined $R < 1$, and following this statement, the right-hand side always converges (property of geometric series). This proves that the series shown in the left-hand side also converges.

We took N to be any number which fulfilled the condition needed, so it is basically an arbitrary real number. Because of this, if

$$\sum_{n=N}^{\infty} |a_N|$$

is convergent, it also means that

$$\sum_{n=0}^{\infty} |a_N|$$

converges as well.

Therefore, the convergence part of the ratio test is proved.

Problem 2

a)

We are going to check if the series below converges.

$$\sum_{k=0}^{\infty} (-1)^k \frac{1}{k+1}$$

This is an alternating series, therefore the most convenient way to check if it converges or not is the Liebniz alternating series test.