The best approximate value of the correlation coefficient that can be calculated using the measurement results is the value R, expressed by the following formula:

$$R = \frac{\sum_{i=1}^{N}\left[(x_i - \bar{x})(y_i - \bar{y})\right]}{\sqrt{\sum_{i=1}^{N}(x_i - \bar{x})^2}\sqrt{\sum_{i=1}^{N}(y_i - \bar{y})^2}}, \quad (4)$$

where $\bar{x}$ and $\bar{y}$ are the average values of the measurement results x_i and yi, respectively:

$$\bar{x} = \frac{1}{N}\sum_{i=1}^{N} x_i \qquad \bar{y} = \frac{1}{N}\sum_{i=1}^{N} y_i\,. \quad (5)$$

The first step is to determine the number ε, which is the root of the following nonlinear equation

$$\alpha = 2 \cdot \Phi(\varepsilon)\ , \tag{7}$$

where Φ (x) is the Laplace function

$$\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_0^x \exp\left(-\frac{t^2}{2}\right) dt\ . \tag{8}$$

In practice, equation (7) is solved using a table of values of the Laplace function. According to the given value of the Laplace function α/2, the corresponding value of the argument ε is extracted from the table. (If there is no numeric value α/2 in the table, linear interpolation should be applied.)

The confidence interval for the correlation coefficient p_{XY} is represented as:

$$th\left(U - \varepsilon/\sqrt{(N-3)}\right) \le \rho_{XY} \le th\left(U + \varepsilon/\sqrt{(N-3)}\right), \tag{9}$$

where U is a value that is related to the approximate value of the correlation coefficient R as follows:

$$U = \frac{1}{2}\ln\left(\frac{1+R}{1-R}\right) \tag{10}$$

The function $th(z)$ is a hyperbolic tangent that can be expressed in terms of the exponential function of the doubled argument:

$$th(x) = \frac{\exp(2x) - 1}{\exp(2x) + 1}. \tag{11}$$