

Solution

One way to prove any given inequality is to work backwards. We can expand and rewrite our inequality into the desired form and prove such a form

$$abc + ab + bc + ac + a + b + c + 1 < 4$$

We are given in the question that $ab + bc + ac = 1$, we can simplify the above into

$$abc + a + b + c < 2$$

It's often helpful to bring all the terms of any inequality to one side

$$2 - a - b - c - abc > 0$$

Now, the more experienced among us might see that this resembles the expansion of $(1-a)(1-b)(1-c)$. In fact, if we substitute $1 = ab + bc + ab$ into the inequality, we have

$$1 - a - b - c + ab + bc + ac - abc > 0$$

which can be factorized into

$$(1 - a)(1 - b)(1 - c) > 0$$