Example

Suppose we wish to solve $x^2 - 3x - 2 = 0$.

We are looking for two numbers which multiply to give -2 and add together to give -3. Never mind how hard you try you will not find any such two numbers. So this equation will not factorise. We need another approach. This is the topic of the next section.

Exercise 1

Use factorisation to solve the following quadratic equations

a) $x^2 - 3x + 2 = 0$ b) $5x^2 = 20$ c) $x^2 - 5 = 4x$ d) $2x^2 = 10x$

e) $x^2 + 19x + 60 = 0$ f) $2x^2 + x - 6 = 0$ g) $2x^2 - x - 6 = 0$ h) $4x^2 = 11x - 6$

3. Solving quadratic equations by completing the square

Example

Suppose we wish to solve $x^2 - 3x - 2 = 0$.

In order to complete the square we look at the first two terms, and try to write them in the form $(\quad)^2$. Clearly we need an x in the brackets:

$(x + ?)^2$ because when the term in brackets is squared this will give the term x^2

We also need the number $-\frac{3}{2}$, which is half of the coefficient of x in the quadratic equation,

$\left(x - \frac{3}{2}\right)^2$ because when the term in brackets is squared this will give the term $-3x$

However, removing the brackets from $\left(x - \frac{3}{2}\right)^2$ we see there is also a term $\left(-\frac{3}{2}\right)^2$ which we do not want, and so we subtract this again. So the quadratic equation can be written

$$x^2 - 3x - 2 = \left(x - \frac{3}{2}\right)^2 - \left(-\frac{3}{2}\right)^2 - 2 = 0$$

Simplifying

$$\begin{aligned}
\left(x - \frac{3}{2}\right)^2 - \frac{9}{4} - 2 &= 0 \\
\left(x - \frac{3}{2}\right)^2 - \frac{17}{4} &= 0 \\
\left(x - \frac{3}{2}\right)^2 &= \frac{17}{4} \\
x - \frac{3}{2} &= \frac{\sqrt{17}}{2} \quad \text{or} \quad -\frac{\sqrt{17}}{2} \\
x &= \frac{3}{2} + \frac{\sqrt{17}}{2} \quad \text{or} \quad x = \frac{3}{2} - \frac{\sqrt{17}}{2}
\end{aligned}$$

We can write these solutions as

$$x = \frac{3 + \sqrt{17}}{2} \quad \text{or} \quad \frac{3 - \sqrt{17}}{2}$$

Again we have two answers. These are exact answers. Approximate values can be obtained using a calculator.

