

FIRST YEAR CALCULUS

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Chapter 2

FUNCTIONS

2.1. Introduction

Let us begin with two very simple examples which everybody can understand.

EXAMPLE 2.1.1. Consider a simple test where there are 4 questions each of which is marked 1 (correct) or 0 (incorrect), and a student is awarded a mark equal to the number of correct answers obtained. Now the possible results that a student can get are the following:

1111	1110	1101	1100
1011	1010	1001	1000
0111	0110	0101	0100
0011	0010	0001	0000

More formally, we may consider a set

$$\mathcal{A} = \{1111, 1110, 1101, 1100, 1011, 1010, 1001, 1000, 0111, 0110, 0101, 0100, 0011, 0010, 0001, 0000\}$$

of all the possible markings, as well as a set $\mathcal{B} = \{0, 1, 2, 3, 4\}$ of the marks awarded. The rule is then given by a function $f : \mathcal{A} \rightarrow \mathcal{B}$, where

$$\begin{aligned} f(1111) &= 4, & f(1110) &= 3, & f(1101) &= 3, & f(1100) &= 2, \\ f(1011) &= 3, & f(1010) &= 2, & f(1001) &= 2, & f(1000) &= 1, \\ f(0111) &= 3, & f(0110) &= 2, & f(0101) &= 2, & f(0100) &= 1, \\ f(0011) &= 2, & f(0010) &= 1, & f(0001) &= 1, & f(0000) &= 0. \end{aligned}$$

EXAMPLE 2.1.2. The set of even natural numbers can be obtained by taking the set $\mathbb{N}$ of all natural numbers and multiplying each of them by 2. More precisely, we can consider a function $f : \mathbb{N} \rightarrow \mathbb{N}$, where $f(x) = 2x$ for every $x \in \mathbb{N}$.