

FIRST YEAR CALCULUS

W W L CHEN

© W W L Chen, 1982, 2008.

This chapter originates from material used by the author at Imperial College, University of London, between 1981 and 1990.

It is available free to all individuals, on the understanding that it is not to be used for financial gain,
and may be downloaded and/or photocopied, with or without permission from the author.

However, this document may not be kept on any information storage and retrieval system without permission
from the author, unless such system is not accessible to any individuals other than its owners.

Chapter 4

SOME SPECIAL FUNCTIONS

4.1. Exponential Functions

In this section we construct a class of functions of the form $f_a : \mathbb{R} \rightarrow \mathbb{R}$, where for every $x \in \mathbb{R}$,

$$f_a(x) = a^x.$$

Here $a > 0$ denotes a positive real constant.

Let us state very carefully what we mean by a^x . We would like to define a^x appropriately so that

$$a^{x+y} = a^x a^y$$

for every $x, y \in \mathbb{R}$. To do so, we must have $a^{x+0} = a^x a^0$. This forces us to write

$$a^0 = 1. \tag{1}$$

Also, it seems reasonable to write

$$a^n = \underbrace{a \dots a}_{n \text{ times}} \quad \text{for every } n \in \mathbb{N}. \tag{2}$$

Next, it is clear that it is necessary to define, for every $p, q \in \mathbb{N}$,

$$y = a^{1/q} > 0 \quad \text{if and only if} \quad y^q = a, \tag{3}$$

and

$$a^{p/q} = (a^{1/q})^p. \tag{4}$$