Considering the units, we have x s. As the 's' on the top line cancels with the 's' on the bottom line, it leaves 'm' the units of distance. That is, the shaded area represents the distance travelled during the acceleration. The symbol given to distance is 's'. Do not confuse this with the abbreviation for seconds.

$\therefore \quad s = 1/2\,(v+u)t$ Equation 2

The third equation is obtained by substituting Equation 1 into Equation 2.

ie $s = 1/2\,[(u+at) + u]t$

$s = 1/2[2u + at]t$

$s = ut + 1/2at^2$ Equation 3

The fourth and final equation of this set is obtained by re-arranging Equation 1 and Equation 2 and then multiplying them together.

ie from Equation 1 $v - u = at$

and from Equation 2 $v + u = \frac{2s}{t}$

$\therefore \quad (v-u)(v+u) = \frac{2s}{t}at$

$v^2 - u^2 = 2as$ Equation 4

Example. A large aircraft has a take-off velocity of 59m/s (about 132 mph). It starts from rest and accelerates uniformly for 30 seconds before becoming airborne.

(a) What is the value of the acceleration in m/s^2?

(b) What take-off distance is required?

(a) $u = 0$

$v = 59$ m/s

$t = 30$ seconds

$a = ?$

using $v = u + at$

$$a = \frac{v-u}{t}$$

$= \dfrac{59-0}{30}$

$= 1.97\ m/s^2$

(b) using s = 1/2(u+v)t

= 1/2(0+59)30

= 885m

FREE FALLING BODIES

Between all masses there is a natural force of attraction. This force depends mainly on the size of the two masses involved and their distance apart (called Newton's universal law of gravitation. Sir Isaac Newton English physicist 1642-1727). When one of the masses is the Earth, then its large mass produces a large force of attraction between it and other masses on or near its surface.

This force, known as a 'gravitational force will cause bodies entering is sphere of influence, to accelerate towards the earth. A body falling freely from a great height will initially be accelerated but wi1l gradually lose this acceleration until it falls with constant velocity, ie zero acceleration. This is known as the body's terminal velocity and depends, amongst other things, on air resistance (drag force).

The body will continue to accelerate will the force of attraction is greater than the drag force, but drag increases as the square of speed. When the drag force eventually reaches the same value as the force of gravity the body has reached its Terminal Velocity. For a streamline shape such as a bomb this is about 1000mph (1600km/h), for a human body it is about 100mph (160km/h).

However, for most general calculations related to free falling bodies, air) resistance is ignored and the body is considered as falling in a vacuum, where its acceleration is uniform having a value of 9.81 m/s2, ie an increase of velocity of 9.81m/s every second. Gravitational acceleration is given the symbol 'g'. The preceding equations 1 to 4 may be used to solve problems involving free falling bodies by substituting 'g' for 'a' in the previous equations.

Example. An aircraft ejector seat is projected vertically upwards, rising freely, after the initial explosive charge has been fired. The initial velocity of the seat is 40m/s as it leaves the aircraft. Determine (a) The maximum height that the seat will reach.

(b) The time taken to reach this height (ignore air resistance).

The ratio of input movement to output movement is called the Velocity Ratio (VR) or 'Movement Ratio'. In figure 10 the effort will move a long way compared to the movement of the load, but the load can be very heavy compared to the input effort.

$$\text{Velocity Ratio (VR)} \quad = \quad \frac{\text{input distance moved}}{\text{output distance moved}}$$

Since both movements occur in the same time this is also the ratio of the input and output velocities. The VR is governed by the physical form of the machine and for a specific machine remains constant for all loads.

Any practical machine will have energy 'losses', often occurring in the form of heat (friction for example). The energy that is 'lost' from the machine is 'gained' by the surroundings. As a result of this the work output from a machine will always be less than the work input.

Comparing 'work-in' to 'work-out' gives the efficiency of the machine.

$$\text{efficiency } (\eta) \quad = \quad \frac{\text{work output}}{\text{work input}}$$

(the symbol η is pronounced eta)

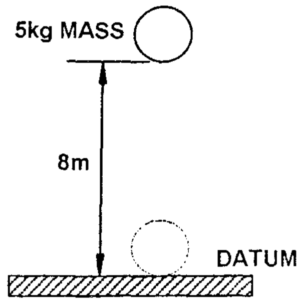
efficiency may be expressed as

$$\eta \quad = \quad \frac{MA}{VR}$$

This must always be less than unity (1), or less that 100% expressed as a percentage, since energy will always be used to overcome friction and other losses.

$$\%\eta = \frac{MA}{VR} \times 100$$

Since the MA varies with loading, the efficiency will reflect this.



Work done = force x distance moved in the direction of the force

= 5kg x 9.81m/s^2 x 8m

= 392.4J

If it were allowed to fall, when the datum is reached the work done becomes the amount of energy the body possesses, ie potential energy = 392.4J.

In this position the mass is capable of doing work and if now allowed to fall freely will strike the ground with 392.4J of energy. Now just before impact all the PE is converted to KE, ie KE = 392.4J. This allows us to calculate the velocity at impact.

Since KE = 392.4J

$$\frac{1}{2}mv^2 = 392.4\text{J}$$

$$v = \sqrt{\frac{392.4 \times 2}{m}}$$

$$v = \sqrt{\frac{392.4 \times 2}{5}}$$

$$v = 12.53 \text{ m/s}$$

Incidentally, on impact the energy would be converted to strain energy of the body and the surface it impacts with plus sound energy.

Example 2: (a) What is the total kinetic energy of a commercial airliner of 200 tons lands at a velocity of 120mph at an actual angle of 5°? (b) Of this total energy how much energy will the brakes have to absorb? (c) How much energy will have to be absorbed by the main oleos (shock absorbers).

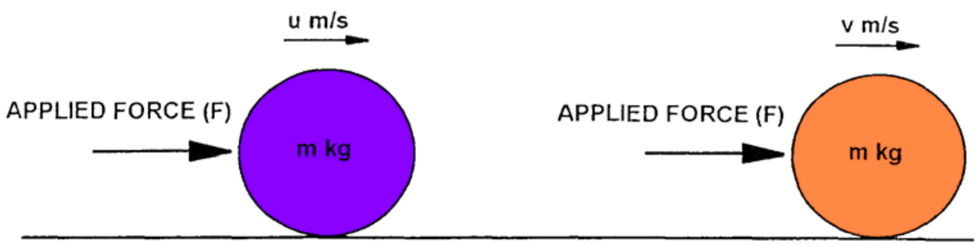


Fig. 18 MOMENTUM OF A SINGLE BODY

initial momentum = mu

final momentum = mv

change of momentum = mv - mu

rate of change of momentum = $\frac{mv - mu}{t}$

Newton's second law states:

$$F \propto \frac{mv - mu}{t}$$

hence $F = \left[\frac{mv - mu}{t}\right] \times \text{constant}$

By a suitable choice of unit, the constant is made to equal unity (1).

Hence $F = \frac{mv - mu}{t}$

or $F = m\left(\frac{v - u}{t}\right)$

now $\frac{v - u}{t} = acceleration$ 'a' m/s^2

Therefore, **F = ma**

If the contact time't' between the bodies involved is very small (st), such as occurs when a body is struck suddenly, then the force applied is known as an 'impulsive force'.

$$\text{Impulsive force} = \frac{\text{change in momentum}}{\text{time taken}}$$

$$= \frac{\text{change in momentum}}{\delta t}$$

When the wheel (called a rotor) is stationary it will behave as any other mass when acted on by an external force. When the wheel is spun it will exhibit properties that are unique to spinning masses. It will retain its original spin axis orientation (called rigidity) and if an external force is applied to it, it will precess (move, in what could be considered to be, an unpredictable movement - though it can be worked out).

The mathematics associated with gyroscopes is fairly complicated, but a knowledge of a few basic terms is necessary. These terms are rigidity, couple, torque, moment of inertia, angular momentum and precession. An understanding of vectors is also essential.

Rigidity

Sometimes called Gyroscopic Inertia, it is the property of a gyro to maintain its axis of rotation with its spin axis pointing to a fixed point — a star infinitely far away in space. So setting a top spinning on a flat surface will mean that is will stand upright, even if the surface on which it rests is moved — provided the top remains spinning. When it stops spinning it will lose its rigidity and fall over.

The amount of rigidity is directly related to the:

(i) Mass of the rotor.

(ii) Speed of the rotor.

(iii) Radius of Gyration or Moment of Inertia of the rotor.

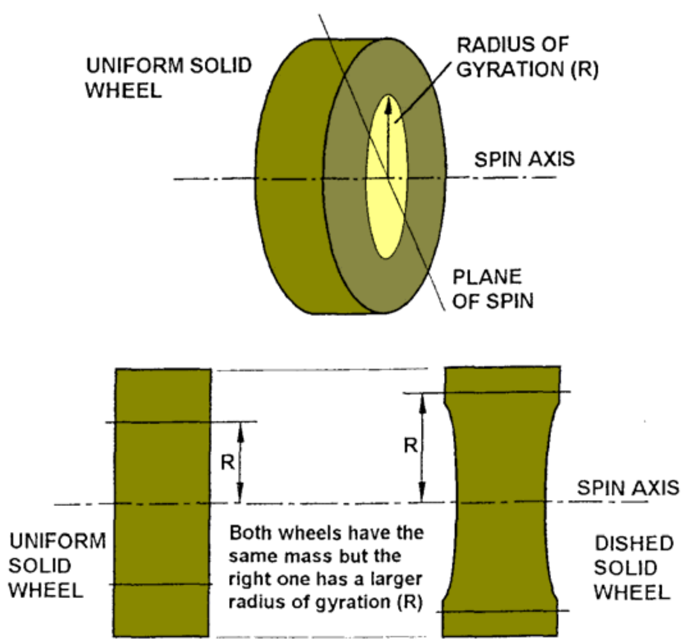


Fig. 21 RADIUS OF GYRATION

The rotor has freedom to rotate about just one axis at right angles to the spin axis so the gyro is said to have "one degree of freedom".

In figure 20 the rotor is suspended in two gimbals, an inner gimbal and an outer gimbal. The rotor is now free to turn relative to the frame about two axes B-B and C-C. The gyro is now said to have "two degrees of freedom".

Gyros are classified as either a displacement gyro or a rate gyro. The displacement gyro measures the amount of movement and the rate gyro measures the rate of change of movement. A rate gyro has a spring or some other restraint mechanism against which rate is measured.

The rotors of the gyros may be driven by air or by dc or ac motors with the electric driven ones usually running faster. The rotor of the motor driven gyro being the actual rotating mass.

Rigidity

As previously discussed, this is the property whereby the spin axis of the gyroscope will remain pointing to a fixed point in space unless acted on by an external force. The property of rigidity is used extensively in aircraft gyros. This means that if the frame is moved (as when the aircraft pitchs or rolls) the gyro rotor axis (if gimballed correctly) will continue to point to a fixed point in space.

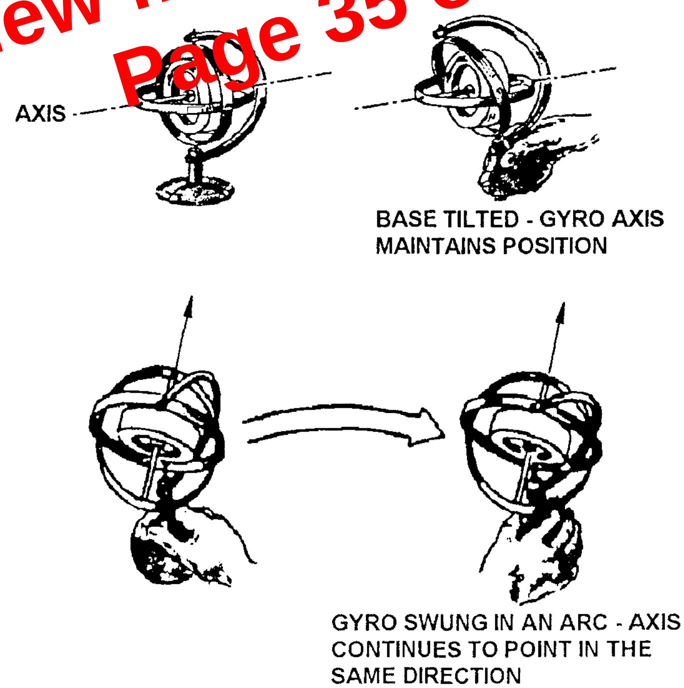


Fig. 26 RIGIDITY