- **Equal matrices:** two matrices $A = [a_{ij}]_{m \times n}$ and $B = [b_{ij}]\, m \times n$ are equal if

  (a) Both have same order

  (b) $a_{ij} = b_{ij} \forall$ i and j

Operations on matrices

- Two matrices can be added or subtracted, if both have same order.
- If $A = [a_{ij}]_{m \times n}$ and $B = [b_{ij}]_{m \times n}$, then

  $A \pm B = [a_{ij} \pm b_{ij}]_{m \times n}$

- $\lambda A = [\lambda\, a_{ij}]_{m \times n}$ where λ is a scalar
- Two matrices A and B can be multiplied if number of columns in A is equal to number of rows in B.

  If $A = [a_{ij}]_{m \times n}$ and $[b_{jk}]_{n \times p}$

  Then $AB = [c_{ik}]_{m \times p}$ where $c_{ik} = \sum_{j=1}^{n} a_{ij} b_{jk}$

  Properties

- If A, B and C are matrices of same order, then

  (i) A+B = B+A

  (ii) (A+B)+C= A+(B+C)

  (iii) A+O = O+A=A

  (iv) A+(-A) = O

39. If $2\begin{bmatrix} x & 5 \\ 7 & y-3 \end{bmatrix} + \begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 7 & 6 \\ 15 & 14 \end{bmatrix}$

Find the valve of $x - y$

40. If A and B are skew symmetric matrices of the same order prove that AB + BA is symmetric matrix.

41. Without expending prove that $\begin{bmatrix} o & p-q & p-r \\ q-p & o & q-r \\ r-p & r-q & o \end{bmatrix} = 0$

42. Evaluate $\begin{bmatrix} 1! & 2! & 3! \\ 2! & 3! & 4! \\ 3! & 4! & 5! \end{bmatrix}$

SHORT ANSWER TYPE QUESTIONS (4 MARKS)

43. If $\begin{vmatrix} a & y & z \\ x & b & z \\ x & y & c \end{vmatrix} = 0$, then prove that $\frac{a}{a-x} + \frac{b}{b-y} + \frac{c}{c-z} = 2$

44. If a ≠ b ≠ c, find the value of x which satisfies the equation

$$\begin{vmatrix} 0 & x-a & x-b \\ x+a & 0 & x-c \\ x+b & x+c & 0 \end{vmatrix} = 0$$

45. Using properties of determinants, show that

$$\begin{vmatrix} a & a+b & a+2b \\ a+2b & a & a+b \\ a+b & a+2b & a \end{vmatrix} = 0$$

46. Find the value of $\begin{vmatrix} \sqrt{13}+\sqrt{3} & 2\sqrt{5} & \sqrt{5} \\ \sqrt{15}+\sqrt{26} & 5 & \sqrt{10} \\ 3+\sqrt{65} & \sqrt{15} & 5 \end{vmatrix}$

47. If $A = \begin{bmatrix} 5 & 3 \\ 12 & 7 \end{bmatrix}$, show that $A^2 - 12A - I = 0$. Hence find A^{-1}.

57. If $A = \begin{bmatrix} 0 & -1 & 2 \\ 4 & 3 & -4 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 0 \\ 1 & 3 \\ 2 & 6 \end{bmatrix}$, then verify that (AB)′ = B′A′

58. If $A = \begin{bmatrix} 0 & -x \\ x & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ $and\ x^2 = -1$

Then show that $(A + B)^2 = A^2 + B^2$

59. Prove that $aI + bA + cA^2 = A^3$, if $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ a & b & c \end{bmatrix}$

60. If $A = \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{bmatrix}$, then find A^3.

61. If $A = \begin{bmatrix} 1 & -1 \\ 2 & 1 \end{bmatrix}$, $B = \begin{bmatrix} a & 1 \\ b & -1 \end{bmatrix}$ and $(A + B)^2 = A^2 + B^2 + 2AB$, find a and b.

62. If $A = \begin{bmatrix} 0 & 2b & c \\ a & b & -c \\ a & -b & c \end{bmatrix}$, then find the value of a, b and c. Such that $A^T A = I$.

63. If $A = \begin{bmatrix} a & b \\ 0 & 1 \end{bmatrix}$, then prove that $A^n = \begin{bmatrix} a^n & b\left(\frac{a^n - 1}{a - 1}\right) \\ 0 & 1 \end{bmatrix}$, for all $n \in N$.

64. If $A = \begin{vmatrix} 1! & 2! & 3! \\ 2! & 3! & 4! \\ 3! & 4! & 5! \end{vmatrix}$, then find A^{-1} and hence prove that $A^2 - 4A - 5I = 0$.

65. Find the value of k, if: $\begin{vmatrix} a+b & b+c & c+a \\ b+c & c+a & a+b \\ c+a & a+b & b+c \end{vmatrix} = k \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$