a particular solution by guessing a form for the solution that includes arbitrary coefficients and then solving for these coefficients by plugging the guess into the differential equation and comparing the resulting expression with the non-homogeneous function.

In this case, we can guess that the particular solution has the same form as x^2, $yp = Ax^3$, where A is an arbitrary coefficient. Plugging this into the differential equation, we get $yp' + yp = Ax^3 + Ax^3 = 2Ax^3$, which is not equal to x^2. This means that our guess needs to be corrected, and we must try a different one.

Another way to guess the form of the particular solution is to note that the non-homogeneous term is a polynomial of degree 2 and that the characteristic equation of the homogeneous part of the differential equation is $r+1 = 0$, with a natural root of $r=-1$. Therefore, we can guess that the solution is $yp = Bx^2e^{-x}$, where B is an arbitrary coefficient. Plugging this into the differential equation, we get $yp' + yp = Bx^2e^{-x} + Bx^2e^{-x} = x^2$, which equals the non-homogeneous function. Therefore, the particular solution of this differential equation is $yp = Bx^2e^{-x}$.

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To find the general solution of the differential equation, we need to add the available resolution of the homogeneous equation ($y = Ce^{-x}$) and the particular solution we found above. Thus, the general answer is $y = (1/3)x^3 + Ce^{-x}$, where C is the constant of integration. It can be determined by applying initial or boundary conditions.

It's worth noting that this differential equation is also known as the Airy equation; it appears as a simplified model in many physical situations, such as the deflection of a beam, the vibration of a string, or the propagation of electromagnetic waves.

6. Find the general solution of the differential equation $y'' + y = 0$

The characteristic equation is $r^2 + 1 = 0$, which gives $r1 = i$ and $r2 = -i$. The general solution is $y = c1\sin(x) + c2\cos(x)$

This differential equation is a second-order linear homogeneous differential equation, which means that the highest derivative in the equation is of second order (in this case, y''), the