

By doing this, we have created a one-to-one correspondence between the elements of the intersection of A and B and the natural numbers, which means that the intersection of A and B is also countable.

3. To prove that the complement of a countable set is uncountable, we can use the fact that the location of real numbers is uncountable and that the accompaniment of a countable set is the set of all real numbers not in that set. Since the set of real numbers is uncountable, and the addition of a countable set is a subset of the group of real numbers, it follows that the complement of a countable set is also uncountable.

The set of real numbers is uncountable, meaning there is no one-to-one correspondence between its elements and the natural numbers. The complement of a group is defined as the set of all features that are not in that set. So, if we take a countable set and find its complement, we will have the set of all real numbers that are not in that countable set.

Since the set of real numbers is uncountable, and the complement of a countable set is a subset of the group of real numbers, it follows that the addition of a countable set is also uncountable.

Given that a subset of a set is smaller than the set itself. And since the complement of a countable set is a subset of the uncountable set of real numbers, it must also be uncountable.

4. To prove that the Cartesian product of two countable sets is countable, we can use the fact that a group is countable if a one-to-one correspondence exists between its elements and the natural numbers. Suppose we have two countable sets, A and B. In that case, we can create a one-to-one correspondence between the details of their Cartesian product  $A \times B$  and the natural numbers by listing all the ordered pairs  $(a, b)$  where  $a$  is an element of A and  $b$  is an element of B in lexicographic order (i.e.,  $(a_1, b_1), (a_1, b_2), \dots, (a_1, b_m), (a_2, b_1), (a_2, b_2), \dots, (a_n, b_n)$ ). This creates a one-to-one correspondence between the elements of  $A \times B$  and the natural numbers, so  $A \times B$  is countable.

The Cartesian product of two sets, A and B, denoted by  $A \times B$ , is the set of all ordered pairs  $(a, b)$  where  $a$  is an element of A and  $b$  is an element of B.