Basic Boolean algebra operations (AND, OR, NOT)

Boolean functions and their truth tables

Number Theory

Definition of a prime number

The Euclidean algorithm for finding the greatest common divisor (GCD) of two numbers

The Fundamental Theorem of Arithmetic

Example of Graph Theory:

Consider the following undirected graph:

A -- B

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| \ |

C -- D

This graph has 4 vertices (A, B, C, D) and 4 edges (AB, AC, AD, BD).

A depth-first search of this graph would start at a vertex (e.g. A) and explore as far as possible along each branch before backtracking. One possible traversal of this graph using depth-first search is: A, C, D, B.

A breadth-first search of this graph would start at a vertex (e.g. A) and explore all vertices at the current depth level before moving on to the next level. One possible traversal of this graph using breadth-first search is: A, B, C, D.

Example of Boolean Algebra:

Consider the Boolean function f(x, y, z) = x'y + y'z.

The truth table for this function is:

Complement: The complement of a set A is the set of elements that are not in A. The complement of A is denoted by A'.

Venn diagrams are graphical representations of sets and set operations. For example, the Venn diagram for the union of two sets A and B is:

A B A U B

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Set theory has many applications, including probability and counting.

Example of Logic and Proofs:

Propositional logic is a branch of logic that deals with statements that are either true or false (called propositions).

Formal proof techniques are used to rigorously prove the validity of statements in propositional logic. Some common proof techniques are:

Direct proof: A direct proof is a proof that starts with the given premises and applies logical reasoning to reach the conclusion.

Proof by contradiction: A proof by contradiction starts by assuming that the negation of the conclusion is true and shows that this leads to a contradiction with the given premises.

Induction: Mathematical induction is a proof technique that is used to prove that a statement is true for all positive integers. It involves proving the base case (usually n = 1) and the induction step (assuming the statement is true for n and showing that it is also true for n + 1).

Logic and proof techniques have many applications, including problem solving and software verification.

Example of Combinatorics:

Combinatorics is the study of finite collections of objects, often with an eye towards counting the number of objects in the collection.

Counting techniques are used to determine the number of possible ways to arrange or select objects from a given set. Some common counting techniques are:

Permutations: A permutation is an arrangement of objects in a specific order. The number of permutations of n objects is n! (n factorial).

Combinatorial Designs

Definition of a combinatorial design

Examples of combinatorial designs (e.g. block designs, Steiner systems)

Applications of combinatorial designs (e.g. experimental design, error-correcting codes)

Group Theory

Definition of a group

Examples of groups (e.g. permutation groups, matrix groups)

Applications of group theory (e.g. symmetry, abstract algebra)

Example of Graph Theory:

A graph is a mathematical structure that represents a set of objects (called vertices or nodes) and the relationships between them (called edges).

There are two types of graphs: directed graphs and undirected graphs. In a directed graph, the edges have a direction (e.g. from vertex A to vertex B). In an undirected graph, the edges have no direction (e.g. between vertex A and vertex B).

Graphs can also be weighted, meaning that each edge has a numerical value associated with it (called a weight).

Graph theory has many applications, including network analysis and computer science.

Example of Combinatorial Designs:

A combinatorial design is a collection of subsets (called blocks) of a larger set, with the property that every two elements of the larger set are contained in exactly one block.

Block designs are a typeof combinatorial design in which the blocks are of the same size. Steiner systems are a type of combinatorial design in which the blocks are of different sizes.

Combinatorial designs have many applications, including experimental design and error-correcting codes.

Example of Group Theory:

A group is a mathematical structure that consists of a set of elements (called the group's elements) and a binary operation (called the group operation) that combines two elements to form a third element.

Example of Probability Theory:

Probability is a measure of the likelihood of an event occurring. It is expressed as a number between 0 and 1, where 0 indicates that the event is impossible and 1 indicates that the event is certain.

There are several axioms and rules that govern probability:

Probability axioms: The probability of an event occurring is a non-negative number and the probability of any event occurring is 1.

Addition rule: The probability of either of two events occurring is the sum of their probabilities.

Multiplication rule: The probability of two events occurring simultaneously is the product of their probabilities.

Probability theory has many applications, including statistical analysis and decision making.

Example of Information Theory:

Information theory is the study of the representation, communication, and processing of information.

Some concepts in information theory include:

Entropy: Entropy is a measure of the amount of uncertainty or randomness in a system. It is often used to quantify the amount of information contained in a message or signal.

Data compression: Data compression is the process of encoding a message or data in a way that takes up less space or bandwidth.

Information theory has many applications, including communication systems and data storage.

Order Theory

Definition of an ordered set

Examples of ordered sets (e.g. real numbers, strings)

Applications of order theory (e.g. sorting algorithms, data structures)

Computability Theory

Definition of computability

Examples of computable and non-computable functions

Applications of computability theory (e.g. theoretical computer science, artificial intelligence)

Topology