# Diminishing Marginal Products Why are factors marginal products decreasing? Consider force xample, the use of capital in an office job

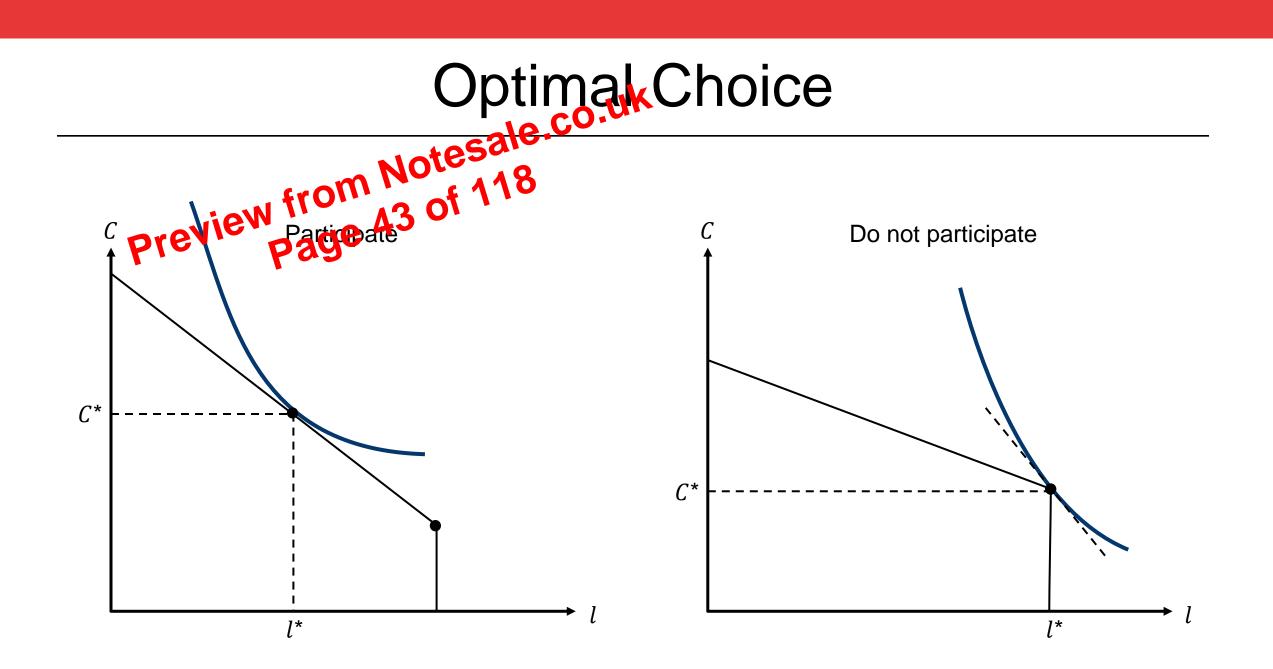
- - Without a computer, it is very difficult to perform many tasks.
  - Giving an office worker a computer has a large effect on that worker's output, so marginal product of capital initially high.
  - And while a more powerful computer may allow the worker to produce more, given the task performed by the worker, the extra computing power is unlikely to raise the worker's output proportionately: the marginal product of capital declines.
- This argument takes as fixed the number of workers, their skills, and state of technology.

- Per-Worker Production Function (cont.)
   If the production function is neoclassical, the per-worker production function has an increasing and concave shape
- Output:  $Y = zNf\left(\frac{K}{N}\right)$
- Marginal product of capital:  $MP_K = \frac{\partial Y}{\partial \kappa} = zf'\left(\frac{K}{N}\right) = zf'(k)$
- $MP_{K}$  is positive but diminishing (in K and k)
- Implies f'(k) is an increasing and concave function of k

## Determinants of Ropulation Growth Malthus argued 01998 that per-capita income and

- consumption affect population growth.
  - Lower consumption per person leads to worse nutrition and health, hence higher death rate and infant mortality, and lower (or negative) population growth rate.
  - Lower income per person induces families to have fewer children they would struggle to support, hence lower birth rate, and lower population growth rate.
  - Higher income and consumption have the opposite effects and raise the population growth rate.
  - Effects are larger when people are close to subsistence.

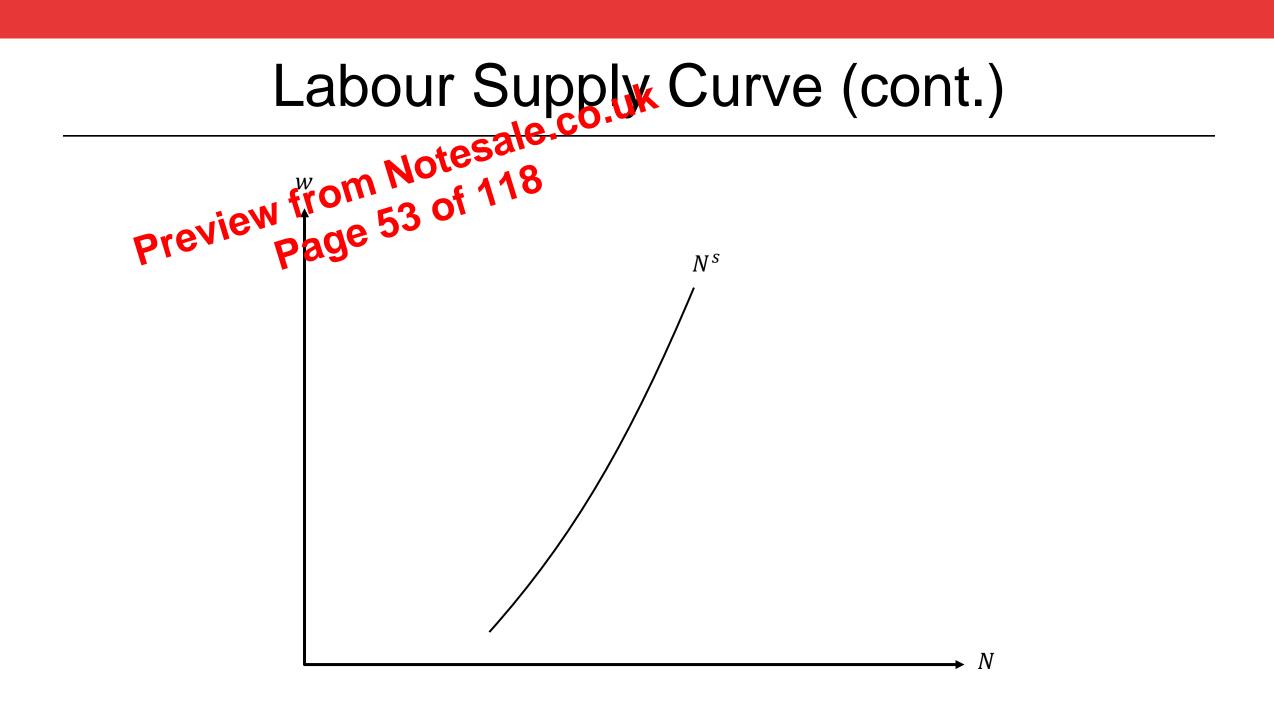
- Agricultural Economy
   Malthus's demographic assumption is particularly relevant for an agricultural economy
  - Land L and labour N are the key factors of production.
  - Land *L* is in fixed supply.
- Neoclassical production function: Y = zF(L, N)
- No capital, so no investment (and ignore government and international trade), hence average consumption per person c is equal to income per worker y = Y/N



### The Effect of Wages on Participation (cont.)

- Higher wage makes budget constraint steeper, pivoting it arounte the point of non-participation.
- Since indifference curve needs to be steeper than budget constraint for non-participation to be chosen, a sufficiently high wage triggers participation in the labour market.
  - No offsetting income effect on the participation decision because higher wage does not make non-participants better off
    - Unless some income comes from a partner who works, in which case higher wages have an income effect on the household labour supply.

- Labour Supply Curve
   The labour supply curve shows the optimal choice of Nsreview Labour Supply curve shows the optimal choice of Nsreview Labour Supply Curve
  - For those participating in the labour market, hours worked increase if the substitution effect is larger than the income effect
  - For those not participating initially, no income effect until some labour is supplied
- N<sup>s</sup> curve upward sloping if SE dominates IE.
  - If everyone participates,  $N^s$  given by equation  $MRS_{l,C} = w$ .



- Equilibrium
  In the goods market: 18
  Fients produce of
  Houseballi
  - Households and the government demand C + G
  - Equilibrium: C + G = Y
  - In the labour market:
    - Firms demand  $N^d$ , determined by  $w = MP_N$
    - Households supply  $N^s$ , determined by  $w = MRS_{LC}$
    - Equilibrium:  $N^d = N^s$
  - Budget constraints:  $C = wN^{s} + \pi T$ ,  $\pi = Y wN^{d}$ , and G = T, combined:  $w(N^d - N^s) = Y - (C + G)$

### Understanding Inequality in Wages

- The last few decades have seen rising income inequality within many econtries. Why?
  - Focus on inequality in wages rather than income inequality more broadly (including capital income) or wealth inequality.
- One important dimension of the rise in wage inequality is the increase in the relative wages of highly skilled workers compared to those with more basic skills.
  - Increase in "college wage premium" in the US and elsewhere.
  - Crucial to debate on the returns to higher education: returns might be higher even if the cost of education has risen.

# Production Using Skilled and Unskilled Labour Separate the supply of labour into high-skilled workers H

- and wirskilled workers N.
  - Supply of *H* related to concept of "human capital" studied later.
- Production function Y = zF(K, N, H) is assumed to be:  $Y = z(K^{\alpha}N^{1-\alpha} + BK^{\alpha}H^{1-\alpha})$ 
  - $\alpha$ , between 0 and 1, is importance of capital relative to labour.
  - z = total factor productivity affects all marginal products.
  - B =skill-biased technology affects marginal product of H.
- Cobb-Douglas production function  $Y = zK^{\alpha}H^{\beta}N^{1-\alpha-\beta}$  not used as cannot build in skill-biased technological change.

- A 14th-Century Pandemic
   Around 1350 Europe North Africa, and Western Asia, are struction a bullonic plague pandemic (the "Black Death").
- This pandemic is believed to have killed more than a third of the population of the affected areas.
- The main economic effect of the pandemic comes from the shortages of labour it creates.

- Factor Payments and Relative Factor Supplies Per-worker production function implies  $Y = zNf\left(\frac{L}{N}\right)$ . Marginal products (using the chain rule):

• Land: 
$$MP_L = \frac{\partial Y}{\partial L} = z \frac{N}{N} f'\left(\frac{L}{N}\right) = z f'(l)$$

• Labour: 
$$MP_N = \frac{\partial Y}{\partial N} = zf\left(\frac{L}{N}\right) - zN\frac{L}{N^2}f'\left(\frac{L}{N}\right) = z(f(l) - lf'(l))$$

- Implies w = y xl, output per worker minus rent times land per worker.
- Depend on relative supply of land to labour l = L/N.
  - $MP_L$  is diminishing in l because f(l) = F(l, 1), so f''(l) < 0.
  - $MP_N$  is increasing in l because derivative of f(l) lf'(l) with respect to *l* is f'(l) - f'(l) - lf''(l) = -lf''(l) > 0.

# Dorfigher Tax Rates Raise More Revenue?

# How to Pay for Public Expenditure? Suppose government needs to pay for public services G.

- Assemble autrel produced using land L and labour N.
  - Land *L* is in fixed supply.
  - Labour N is labour supply of a representative household.
  - Production function: Y = aN + bL
    - Linear in both L and N (not neoclassical, but useful for illustration).
    - Marginal products are  $MP_N = a$  and  $MP_L = b$  (where a and b are positive constants).
  - Pre-tax wage w and rent x determined in competitive markets:

$$w = MP_N = a$$
 ,  $x = MP_L = b$ 

## Switch from Taxes on Wages to Taxes on Rents • Initially only wage income taxed: $a\tau_w N = G$ and $\tau_x = 0$

- Optimial choice of C and l = h N at tangency of budget constraint and indifference curve:  $MRS_{l,C} = a(1 - \tau_w)$ .
  - On budget constraint, so  $C + a(1 \tau_w)l = a(1 \tau_w)h + bL$ .
- Switch to tax on rents:  $b\tau_x L = G$  and  $\tau_w = 0$ 
  - Assuming bL > G, so enough rental income to tax.
- Budget constraint steeper; shifts down at l = h.
- Observe that same choice of C and l remains affordable:
  - Original *C* and *l* satisfy  $C + al = ah + b(1 \tau_x)L$ .

• Because 
$$a\tau_w(h-l) = a\tau_w N = G = b\tau_x L$$
.

## Production Function

- Focusing on capital and labour as the factors of production and ignoring land, the production function is: Y = zF(K, N)
- Gross domestic product (GDP) is *Y*, the labour force is *N* (number of workers), the capital stock is *K*, and total factor production (TFP) is *z*
- The production function F(K, N) is neoclassical
  - Diminishing returns to capital
- For example, the Cobb-Douglas production function:  $Y = zK^{\alpha}N^{1-\alpha}$
- Parameter  $\alpha$  estimated using capital share of income

- Capital Accumulation
   Changes in the amount of capital accumulated per worker are explained by the difference between:
  - szf(k) = amount of saving, and hence investment, per worker
    - Saving rate s multiplied by per-worker production function y = zf(k).
  - (d+n)k = amount of investment per worker needed to sustain same level of capital per worker next year
    - A fraction d of all capital depreciates, so an amount of capital per worker dk must be replaced to keep the capital per worker unchanged.
    - The number of workers increases by a percentage n each year, so if existing workers use capital k each, there needs to be investment nk per existing worker to give future workers the same capital k each.

