13. Inverse operations

Suppose we pick a base, 2 say.

Suppose we pick a power, 8 say.

We will now raise the base 2 to the power 8, to give 2^8.

Suppose we now take logarithms to base 2 of 2^8.

We then have

$$\log_2 2^8$$

Using the laws of logarithms we can write this as

$$8 \log_2 2$$

Recall that $\log_a a = 1$, so $\log_2 2 = 1$, and so we have simply 8 again, the number we started with.

So, raising the base 2 to a power, and then finding the logarithm to base 2 of the result are inverse operations.

Let's look at this another way.

Suppose we pick a number, 8 say.

Suppose we find its logarithm to base 2, to evaluate $\log_2 8$.

Suppose we now raise the base 2 to this power: $2^{\log_2 8}$.

Because $8 = 2^3$ we can write this as $2^{\log_2 2^3}$. Using the laws of logarithms this equals $2^{3\log_2 2}$ which equals 2^3 or 8, since $\log_2 2 = 1$. We see that raising the base 2 to the logarithm of a number to base 2 results in the original number.

So raising a base to a power, and finding the logarithm to that base are inverse operations. Doing one operation, and then following it by the other, we end up where we started.

Example
Suppose we are working in base e. We can pick a number x and evaluate e^x. If we follow this by taking logarithms to base e we obtain

$$\ln e^x$$

Using the laws of logarithms this equals

$$x \ln e$$

but $\ln e = 1$ and so we are left with simply x again. So, raising the base e to a power, and then finding logarithms to base e are inverse operations.

Example
Suppose we are working in base 10. We can pick a number x and evaluate 10^x. If we follow this by taking logarithms to base 10 we obtain

$$\log 10^x$$

Using the laws of logarithms this equals

$$x \log 10$$

but $\log 10 = 1$ and so we are left with simply x again. So, raising the base 10 to a power, and then finding logarithms to base 10 are inverse operations.