Note:-

If all the limits of double integrals are numbers, then the integrals are indentified using rectangle box, and any order of integration (x first y second or y first x second) can be followed, both will give same answer.

Problem:-02

Evaluate $\int_2^3 \int_1^2 \frac{1}{xy}\, dx\, dy$

Solution:-

$$I = \int_{y=2}^{y=3} \left[\int_{x=1}^{x=2} \frac{1}{xy}\, dx\right] dy$$

[i.e region corresponding to the above integral is rectangular region bounded by x=1, x=2, y=2 and y=3]

$$= \int_2^3 \frac{1}{y} (\log x)_{x\,=\,1}^{x\,=\,2}\, dy$$

$$= \int_2^3 \frac{1}{y} (\log 2 - \log 1)\, dy$$

$$= \int_2^3 \frac{1}{y} (\log 2)\, dy \;=\; \log 2\; [\log y]_{y\,=\,2}^{y\,=\,3}$$

= log 2 [log 3 – log 2]

= log2 . log 3/2

= log 2*3/2

= log 3.

Problem:- 3

Evaluate: $\int_{x=0}^{x=5} \int_{y=0}^{y=x^2} x(x^2 + y^2)\, dx\, dy$

Problem:-05

Evaluate $\int_0^1 \int_0^{\sqrt{1+x^2}} \frac{dx\,dy}{1+x^2+y^2}$

Solution:

Since y=0 and $y=(1+x^2)^{1/2}$, therefore the order of integration with respective y first and x second

Let I = $\int_0^1 \int_0^{\sqrt{1+x^2}} \frac{dx\,dy}{1+x^2+y^2}$

$$= \int_0^1 \int_0^{\sqrt{1+x^2}} \frac{1}{\left(\sqrt{1+x^2}\right)^2+y^2}\, dy\, dx$$

$$= \int_0^1 \left[\frac{1}{\sqrt{1+x^2}} tan^{-1}\left(\frac{y}{\sqrt{1+x^2}}\right)\right]_0^{\sqrt{1+x^2}} dx$$

$$= \int_0^1 \left[\frac{1}{\sqrt{1+x^2}} tan^{-1}(1) - tan^{-1}(0)\right] dx$$

$$= \int_0^1 \left[\frac{1}{\sqrt{1+x^2}} \frac{\pi}{4}\right] dx$$

$$= \frac{\pi}{4} \int_0^1 \left[\frac{1}{\sqrt{1+x^2}}\right] dx$$

$$= \frac{\pi}{4} [sinh^{-1}(x)]_0^1$$

$$= \frac{\pi}{4} [sinh^{-1}(1) - sinh^{-1}(0)]$$

$$= \frac{\pi}{4} [sinh^{-1}(1) - 0]$$

$$= \frac{\pi}{4} [\log(1+\sqrt{2})].$$